THEORETICAL STUDY OF HIGHER ORDER NONCLASSICALITY IN INTERMEDIATE STATES

Synopsis of the thesis to be submitted in fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

By

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1 Introduction

A state of light is called nonclassical, if its Glauber Sudarshan P function is negative or more singular than a delta function [1]. In these situations quasi probability distribution P is not accepted as classical probability and thus one can not obtain an analogous classical state. For example, squeezed state and antibunched state are well known nonclassical states. These two lowest order nonclassical states have been studied since long but the interest in higher order nonclassical states is relatively new. Possibilities of observing higher order nonclassicalities in different physical systems have been investigated in recent past [2]-[16]. For example, i) higher order squeezed state of Hong Mandel type [2]-[4], ii) higher order squeezed state of Hillery type [5], [6], iii) higher order subpoissonian photon state [7]-[9] and iv) higher order antibunched state [10]-[16] are recently studied in different physical systems. All these interesting recent studies and their potential applications in quantum optics, quantum information, quantum computing and other fields motivated us to study the higher order nonclassical states of light in detail.

Among the different class of states which shows nonclassicality. Intermediate states form an interesting group. An intermediate state is a quantum state which reduces to two or more distinguishably different states (normally, distinguishable in terms of photon number distribution) in different limits. In 1985, such a state was first time introduced by Stoler etal. [17]. To be precise, they introduced Binomial state (BS) as a state which is intermediate between the most nonclassical number state $|n\rangle$ and the most classical coherent state $|\alpha\rangle$. Since the introduction of BS, the intermediate states have attracted considerable attention of physicists. Consequently, different properties of binomial states have been studied [18]-[22]. In these studies it has been observed that the nonclassical phenomena (such as, antibunching, squeezing and higher order squeezing) can be seen in BS. This trend of search for nonclassicality in Binomial state, continued in nineties. In one hand, several versions of generalized BS have been proposed [19]-[27] and in the other hand, people went beyond binomial states and proposed several other form of intermediate states (such as, excited binomial state (EBS) [21], odd excited binomial state (OEBS) [22], negative hypergeometric state (NHS) [23], reciprocal binomial state (RBS) [24], photon added coherent state (PACS) [25] etc.) and hypergeometric state (HS) $[27]^1$. The studies in the last century were mainly limited to theoretical predictions of lower order nonclassicalities but the recent developments in the experimental techniques made it possible to experimentally verify some of those theoretical predictions. For example, we can note that, as early as in 1991 Agarwal and Tara [25] had introduced PACS but the experimental generation of the state has happened only in recent past when Zavatta, Viciani and Bellini [26] succeed to produce it in 2004. This exper-

¹Mathematical definitions of these states are provided in Apendix A

imental observation, several existing reports of lower order nonclassicalities in intermediate states and the intrinsic nonclassical character of the intermediate states have motivated us to study the possibilities of observing higher order nonclassicalities in intermediate states. Present work aims to provide a clear understanding of higher order nonclassical states in general with specific attention towards intermediate states and many wave mixing processes [8], [12]-[16], [28]-[32]. Chronological development of the subject is provided in Table 1. Our achievements appear in the lower part of the Table 1. The results of the present work, which are reported in [8], [12], [28]-[30], will be discussed in detail in the proposed thesis.

In this synopsis we try to provide a brief sketch of the proposed doctoral thesis. To do so we have categorized this synopsis in 6 sections including introduction. Each section contain brief summary of individual chapters of the thesis. To be precise, first chapter of the proposed thesis will provide an introduction to the subject and major content of that chapter is accepted for publication in our review article [7]. Section 2, briefly describes the content of chapter 2 of the proposed thesis. Here we describe the possibility of observing higher order antibunching in intermediate states and in some simple nonlinear optical phenomena. The results reported in this chapter are published in [12],[29]-[30]. In section 3 we report that a generalized notion of higher order nonclassicality (in terms of higher order moments) can be introduced. Under this generalized framework of higher order nonclassicality, conditions of higher order squeezing and higher order subpoissonian photon statistics are derived. These simplified criteria play the central role in the present work as these criteria are used in next chapter to study the presence of higher order nonclassicalities in different intermediate states. The results summarized in section 3 are published in [8] and they will be described in detail in the chapter 3 of the proposed thesis. The section 4 provides a glimpse of the work to be reported in detail in the chapter 4 of the proposed thesis. In this section we briefly describe our work on the possibilities of observing higher order squeezing and higher order subpoissonian photon statistics in different intermediate states. Corresponding results are partly published [7],[8] and partly communicated. Section 5 shows reduction of quantum phase fluctuations (U parameter), which is a stronger criterion of nonclassicality (compared to the lowest order antibunching), is possible in intermediate states. Corresponding results are published in [28] and will be described in detail in chapter 5 of the thesis. Section 6 briefly summarizes the content of the concluding chapter of the proposed thesis. The conclusions are derived from the earlier chapters of the thesis (i.e. from the works reported in [7],[8],[12],[28]-[30]). Limitations of the present work & scope of future works are also described here. Definition of different intermediate states is given in Appendix A, mathematical criteria of different kind of nonclassicality are shown in Appendix B and the results related to the study of higher order nonclassicalities in binomial state are provided in Appendix C as an example.

Year	${f Intermediate}\ {f states}$	Author	Lower order nonclassicality studied	Higher order nonclassicality studied	Remarks
1985	BS	Stoler [17]	Squeezing	-	Introduce first intermediate
1000	20		5 queening		state (Binomial state)
1985	-	Hong & Mandel	_	HOS	Introduce Higher order
		[2]			squeezing.
1990	-	C T Lee [10]	-	HOA	Introduce Higher order
					nonclassicality in general.
1991	PACS	Agrawal & Tara [25]	Squeezing, sub poissonian statistics	-	Photon added coherent state (PACS)
1994	BS	Vidiella et al [18]	Squeezing, sub poissonian statistics	HOS(4th order)	$\operatorname{Binomial\ state}(\operatorname{BS})$
1996	HS	Fu & Sasaki [27]	Squeezing, Antibunching, sub poissonian statistics	-	One parameter generalization of Binomial State(BS)
1997	GBS	Roy & Roy [20]	Squeezing, Antibunching	-	Generalized Binomial state (GBS)
1998	NBS	Barnett [33]	-	-	Introduce Negative Binomial state (NBS)
1998	RBS	Baseia [24]	-	-	Generation Scheme
1998	NHS	Fan & Liu [23]	Squeezing, sub	-	Introduce Negative
			poissonian statistics		${ m hypergeometric\ state}$
2000	EBS	Wang & Fu [21]	Squeezing, sub poissonian statistics	-	Excited Binomial state (EBS)
2002	OEBS	Obada & Darwish [22]	Squeezing, sub poissonian statistics	-	Odd Excited Binomial state (OEBS)
2002	-	Erenso et al [11]	-	НОА	Experimental observation of HOA
2003	RBS	Valverde [34]	-	-	Generation scheme
2004	PACS	Zavatta et al [26]	-	-	Experimental generation
2005	-	Gupta & Pathak [32]	-	НОА	HOA is not a rare phenomenon
2006	-	Hari Prakash & Mishra [9]	-	HOS, HOSPS	
2006	-	Pathak [16]	-	НОА	Mathematical criterion of nth order Antibunching
2007	NLSS	Darwish [35]	Squeezing, sub poissonian statistics	-	Another class of minimum uncertainty states
2008	BS, GBS, NBS, HS, RBS and PACS	Verma, Sharma & Pathak [12]	-	НОА	Higher order antibunching (Generalized) in intermediate states
2008	PACS	Duc & Noh [6]	-	HOS, HOSPS	4th order and 6th order calculations
2008	-	Verma & Pathak [30]		HOA & HOSPS	Many wave mixing processes
2009	BS, GBS, NBS, HS and PACS	Verma & Pathak [28]	-	-	Reduction of quantum phase fluctuations
2010	BS, NLSS	Verma & Pathak [8]	-	HOS, HOSPS	Generalized structure of higher order nonclassical states

Table 1: Chronological development in the study of nonclassical properties of intermediate states.

2 Higher order antibunching in intermediate states

Since the introduction of binomial state as an intermediate state, different intermediate states have been proposed in the literature. Different nonclassical effects have also been reported in these intermediate states. But till 2007 higher order antibunching (HOA) was predicted in only one type of intermediate state, which is known as shadowed negative binomial state. In recart past other members of our group have shown that the HOA can be seen in optical processes [13] and it is not a rare phenomenon. To establish that claim further, here we have shown that the HOA can be seen in different intermediate states also, such as binomial state, reciprocal binomial state, hypergeometric state, generalized binomial state, negative binomial state and photon added coherent state. We have studied the possibility of observing the higher order antibunching in different limits of intermediate states. The effects of different control parameters on the depth of non classicality have also been studied in this connection and it has been shown that the depth of nonclassicality can be tuned by controlling various physical parameters. We have observed the following:

- 1. Most of the intermediate states studied in this work show HOA [12]. But all the intermediate states are not higher order antibunched (for example, photon subtracted coherent state is always higher order bunched).
- 2. An intermediate state which shows HOA may also show higher order bunching or higher order coherence for particular values of the control parameters (e.g. α , N, m etc.). The depth of nonclassicality of a higher order antibunched state is also varies with these parameters.
- 3. As far as HOA is concerned there does not exist any common characteristics among the different intermediate states but most of them show HOA.
- 4. A potential scheme for the experimental observation of HOA in intermediate state can be introduced (see [12]).

3 Generalized structure of higher order nonclassical states

A generalized notion of higher order nonclassicality (in terms of higher order moments) is introduced. Under this generalized framework of higher order nonclassicality, conditions of higher order squeezing (HOS) and higher order subpoissonian photon statistics (HOSPS) are derived. A simpler form of the HOS (Hong-Mandel type) criterion is derived under this framework by using an operator ordering theorem introduced by Pathak in [31]. It is

also generalized for multi-photon Bose operators of Brandt and Greenberg type. Similarly, condition for HOSPS is derived by normal ordering of higher powers of number operator. Further, with the help of simple density matrices, it is shown that the HOA and HOSPS are not the manifestation of the same phenomenon and consequently it is incorrect to use the condition of HOA as a test of HOSPS. It is also shown that the HOA and HOSPS may exist even in absence of the corresponding lower order phenomenon. In literature HOA and HOSPS have been used as synonymous (see [6]). Our observations establish that it is incorrect to use the condition of HOA as a test of HOSPS. Present study [8] is the first one of its kind in which rigorous attempts have been made to understand the mutual relationship between different higher order nonclassical states. The effort is successful to provide an insight into the mutual relations between the well known nonclassical states and opens up a possibility of similar work in broader class of nonclassical states. The simpler framework provided for the study of possibilities of observing Hong Mandel squeezing is also expected to be useful in the future works.

4 Higher order squeezing and higher order subpoissonian photon statistics in intermediate states

Simpler criteria for the Hong-Mandel higher order squeezing (HOS) and higher order subpoissonian photon statistics (HOSPS) provided by us [8] are used here to study the possibilities of observing HOSPS and HOS in different intermediate states. It is shown that these states may satisfy the condition of HOS and HOSPS. It is also shown that the depth of nonclassicality and the region of nonclassicality can be controlled by controlling various parameters related to intermediate states. Further, we have analyzed the mutual relation between different signatures of higher order nonclassicality with respect to these intermediate states. Binomial state, nonlinear first order excited squeezed state (NLESS), nonlinear vacuum squeezed state (NLVSS) and other intermediates states mentioned earlier are used as examples of quantum states and it is shown that these states may show higher order nonclassical characteristics. It is observed that the Binomial state which is always antibunched, is not always higher order squeezed and NLVSS which shows higher order squeezing does not show HOSPS and HOA. The opposite is observed in NLESS and consequently it is established that the HOSPS and HOS are two independent signatures of higher order nonclassicality. The relation between HOA, HOSPS and HOS is investigated in detail and certain interesting observations in this regard have been reported. For example,

1. BS always shows HOA and HOSPS but it does not show HOS for all values of p. So we conclude that existence of HOSPS does not guarantee the existence of HOS. This is

consistent with the corresponding observations in lower order.

- 2. NLVSS which shows higher order squeezing does not show HOSPS and HOA. The opposite is observed in NLESS and consequently it is established that the HOSPS and HOS are two independent signatures of higher order nonclassicality.
- 3. GBS may show signature of HOSPS in absence of HOS. Earlier we have shown that NLVSS shows HOS in absence of HOSPS. Consequently it is established that the HOSPS and HOS of same order are independent phenomenon.

5 Reduction of quantum phase fluctuations in intermediate states

Recently other members of our group had shown that the reduction of the Carruthers-Nieto symmetric quantum phase fluctuation parameter (U) with respect to its coherent state value corresponds to an antibunched state, but the converse is not true [32]. Consequently reduction of U is a stronger criterion of nonclassicality than the lowest order antibunching. Here we have studied the possibilities of reduction of U in intermediate states by using the Barnett Pegg formalism. We have shown that the reduction of phase fluctuation parameter Ucan be seen in different intermediate states. It is also shown that the depth of nonclassicality can be controlled by various parameters related to intermediate states. Further, we have provided specific examples of antibunched states, for which U is greater than its poissonian state value. Our observations are as follows:

- 1. Most of the intermediate states described above show reduction of U with respect to its coherent state value²[28]. This establishes that the intermediate states can satisfy the stronger criterion of nonclassicality compared to the criterion of usual antibunched state. This is consistent with our earlier observation of higher order antibunching of intermediate states.
- 2. In binomial state, U parameter does not satisfy the criterion for all values of probability p, while in chapter 2 we have shown that Binomial state is always antibunched up to any order. Thus for higher values of p (i.e when p is close to 1) it is higher order antibunched but do not satisfy the criterion laid down on the basis of quantum phase fluctuations. Similar phenomenon is also observed in HS.
- 3. Other members of our group had earlier reported that reduction of quantum phase fluctuation means antibunching but the converse is not true. This is the first time when

²In reciprocal binomial state, we have not observed this phenomenon.

an example of such a state which is antibunched but does not show reduction of quantum phase fluctuation with respect to coherent state, is found.

- 4. Further from the study of phase properties of GBS and HS we have learnt that the reduction of quantum phase fluctuation mean antibunching but does not essentially mean higher order antibunching and therefore, it is not essential that these two stronger conditions of nonclassicality appear simultaneously.
- 5. In connection to PACS we have observed that the more photon are added to coherent state the more nonclassical the PACS, is as far as the depth of nonclassicality associated with quantum phase fluctuation is concerned. This particular characteristic has also been reflected in higher order antibunching.

6 Conclusions

In the present work we have observed different interesting facts regarding higher order nonclassical states of light in general and possibilities of observing higher order nonclassicality in different physical systems (intermediate states and many wave mixing processes) in particular. We can conclude the present work with the following observations.

- 1. Normally criterion of a particular kind of nonclassicality is presented through an inequality and once an inequality is established several technical reports appears with the observation that a particular physical system satisfies this criterion. Here we have shown that arbitrary large number of such higher order inequalities can be established and consequently reports of satisfaction of particular nonclassical criterion in particular system does not provide much new Physics unless the criterion itself has a very specific physical meaning. Such a meaning is associated with the condition of HOA, as potential single photon sources are expected to satisfy the condition of HOA. HOA is observed by us in several physical systems. Their quality as potential SPS is compared in the present work.
- 2. A clear definition of higher order nonclassicality is provided and it is shown that Hillery type squeezing are not higher order according to this definition. It is also shown that the criteria of HOSPS and Hong Mandel type of higher order squeezing can be derived from a single framework. It is shown that the lower order antibunching, HOA and HOSPS appear in novel regimes. But in literature HOA and HOSPS have been used as synonymous. Our observations establish that it is incorrect to use the condition of HOA as a test of HOSPS.

3. The work provides an insight into the mutual relations between the well known nonclassical states and opens up a possibility of similar work in broader class of nonclassical states. The simpler framework provided here for the study of possibilities of observing Hong Mandel squeezing is also expected to be useful in the future works. It is shown that the theoretical predictions of the present work can be verified experimentally. It is also observed that intermediate states (e.g. BS, GBS, NBS, HS & PACS) show reduction of U with respect to its coherent state value but RBS does not show reduction of U. This establishes that the intermediate states can satisfy the stronger criterion of nonclassicality compared to the criterion of usual antibunched state.

6.1 Limitations and scope of future works

The major limitation of the present work lies in the fact that there exist several theoretical proposals for generation of intermediate states but apart from PACS no other intermediate states have been experimentally realized so far. We believe that the theoretical predictions of the present work, recent experimental generation of PACS and the possibility of using intermediate states for quantum communication will motivate experimentalist to create these quantum states in the laboratory. Here we would like to note that the present study is the first one of its kind in which rigorous attempts have been made to understand the mutual relationship between different higher order nonclassical states of light. The effort is successful to provide an insight into the mutual relations between the well known nonclassical states and opens up a possibility of similar work in broader class of nonclassical states. The simpler framework provided for the study of possibilities of observing Hong Mandel squeezing is also expected to be useful in the future works. The scope of future work is manifold. At present strong possibilities of future work appears in the following:

- 1. Applications of higher order antibunched light in quantum cryptography. Specially in the realization of single photon sources and quantum random number generator. They may used in continuous variable quantum cryptography too.
- 2. Experimental verification of the theoretical results of the present work.
- 3. Use of squeezed light in quantum teleportation.
- 4. Study of higher order nonclassicalities in other quantum states of relevance (e.g. superposition states).

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List of publications during PhD thesis work:

- 1. Pathak A and Verma A, "Recent developments in the study of higher order nonclassical states", Indian Journal of Physics, in press, 2010.
- Verma A and Pathak A, "Generalized structure of higher order nonclassicality", Physics Letters A, vol.374, pp 1009-1020, 2010
- 3. Verma A and Pathak A, "Reduction of quantum phase fluctuations in intermediate states" Phys. Lett A, vol 373, pp 1421, 2009..
- 4. Verma A, Sharma N K and Pathak A, "Higher order antibunching in intermediate states", Phys. Lett A, vol. 372, pp. 5542, 2008.
- 5. Verma A and Pathak A, "Higher order squeezing and Higher order sub poissonian photon statistics in intermediate states" Communicated
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APPENDIX-A

Definition of different Intermediate states is given below.

Year of	Intermediate	Author	Definition
Introduction	states		
1985	BS	Stoler [17]	Binomial state is defined as $ p, M\rangle = \sum_{n=0}^{M} B_n^M n\rangle$
			$ p, M\rangle = \sum_{n=0}^{N} B_n n\rangle \\ = \sum_{n=0}^{M} \sqrt{MC_n p^n (1-p)^{M-n}} n\rangle , \ 0 \le p \le 1$
1001	DAGG		
1991	PACS	Agrawal & Tara	Photon added coherent state (PACS) is defined as
		[25]	$ \alpha,m\rangle = \frac{\exp(-\frac{ \alpha ^2}{2})}{\sqrt{L_m(- \alpha ^2)m!}} \sum_{n=0}^{\infty} \alpha^n \frac{\sqrt{(m+n)!}}{n!} n+m\rangle$
			where $L_m(x)$ is Lauguere polynomial of <i>mth</i> order.
1996	RBS	Barnett et	Reciprocal binomial state (RBS) is defined as
		al[24]	
			$ \phi angle = rac{1}{\aleph} \sum_{k=1}^{N} \left({}^{N}C_{k} ight)^{-1/2} e^{ik(heta-\pi/2)} k angle$
			k=0
			where \aleph is a normalization constant.
1996	HS	Fu & Sasaki	Hypergeometric state (HS) is defined as
		[27]	$ L, M, \eta\rangle = \sum_{n=0}^{M} H_n^M(\eta, L) n\rangle$ where
			$ \begin{aligned} H_n^M(\eta, L) &= \left[\begin{pmatrix} L\eta \\ n \end{pmatrix} \begin{pmatrix} L(1-\eta) \\ M-n \end{pmatrix} \right]^{\frac{1}{2}} \begin{pmatrix} L \\ M \end{pmatrix}^{-\frac{1}{2}}, \text{ with} \\ 0 < \eta < 1 \text{ and } L \ge max \left\{ M\eta^{-1}, M(1-\eta)^{-1} \right\}. \end{aligned} $
1007	CDC	D (D [90]	
1997	GBS	Roy & Roy [20]	Generalized Binomial state (GBS) is defined as $\sum_{k=1}^{N} \sqrt{\left(\frac{N}{2}\right)^{k}} = 1$
			$ N, \alpha, \beta\rangle = \sum_{n=0}^{N} \sqrt{\omega(n, N, \alpha, \beta)} n\rangle \text{ where}$
			$\omega(n, N, \alpha, \beta) = \frac{N!}{(\alpha + \beta + 2)_N} \frac{(\alpha + 1)_n (\beta + 1)_{N-n}}{n! (N-n)!} \text{ with } \alpha, \beta > -1,$
			$n = 0, 1,, N$, and $(a)_0 = 1$ $(a)_n = a(a+1)(a+n-1)$
1998	NBS	Barnett [33]	NBS is defined as $ \eta, M\rangle = \sum_{n=M}^{\infty} C_n(\eta, M) n\rangle$ where
			$C_n(\eta, M) = \left[\begin{pmatrix} n \\ M \end{pmatrix} \eta^{M+1} (1-\eta)^{n-M} \right]^{1/2}, \ 0 \le \eta \le 1 \text{ and } M$
			is a non-negative integer.
2007	NLSS	Darwish [35]	Nonlinear squeezed states (NLSS) are of two types, named as
			nonlinear vacuum squeezed state (NLVSS) and nonlinear first
			order excited squeezed state (NLESS).
			NLVSS is defined as $ \psi\rangle_V = N \sum_{n'=0}^{\infty} \frac{\sqrt{(2n')!}}{n'![f(2n')]!} \left[\frac{\xi_1}{2}\right]^{n'} 2n'\rangle$
			with $ N ^{-2} = \sum_{n'=0}^{\infty} \frac{(2n')!}{(n'!)^2 [f(2n')!]^2} \left[\frac{\xi_1}{2}\right]^{2n'}$ and
			NLESS as $ \phi\rangle_E = N' \sum_{n'=0}^{\infty} \frac{\sqrt{(2n'+1)!}}{n![f(2n'+1)]!} \left[\frac{\xi_1}{2}\right]^{n'} 2n'+1\rangle$ with
			$ N' ^{-2} = \sum_{n'=0}^{\infty} \frac{(2n'+1)!}{(n'!)^2 [f(2n'+1)!]^2} \left[\frac{\xi_1}{2}\right]^{2n'}$

APPENDIX-B

Higher order nonclassicality: simplified criteria

Commonly, second order moment (standard deviation) of an observable is considered to be the most natural measure of quantum fluctuation associated with that observable and the reduction of quantum fluctuation below the coherent state (poissonian state) level corresponds to lowest order nonclassical state. For example, an electromagnetic field is said to be electrically squeezed field if uncertainties in the quadrature phase observable X reduces below the coherent state level (i.e. $(\Delta X)^2 < \frac{1}{2}$) and antibunching is defined as a phenomenon in which the fluctuations in photon number reduces below the Poisson level (i.e. $(\Delta N)^2 < \langle N \rangle$) . In analogy, if we consider an arbitrary quantum mechanical operator A and a quantum mechanical state $|\psi\rangle$ then the state $|\psi\rangle$ is *nth* order nonclassical with respect to the operator A if

$$(\Delta A)^n_{|\psi\rangle} < (\Delta A)^n_{|poissonian\rangle},\tag{1}$$

where $(\Delta A)^n$ is the *n*th order moment defined as

$$\langle (\Delta A)^n \rangle = \sum_{r=0}^n {}^n C_r (-1)^r \overline{A^r} \ \bar{A}^{n-r}.$$
 (2)

If A is a field operator then it can be expressed as a function of creation and annihilation operators a^{\dagger} and a and consequently further simplification of (1) is possible. Thus we can say that for a higher order nonclassical state *nth* order moment (n > 2) of the observable A is smaller than that of a coherent state.

It is clear from (2) that the problem of finding out the *nth* order moment of the operator A essentially reduces to a problem of operator ordering (normal ordering) of A^r . In the present work we have used normal ordered form of X^r and N^r to obtain simplified conditions for HOS and HOSPS. Since these criteria form the main mathematical frame work of the thesis, we have briefly described these simplified criteria in the following subsections.

Simplified condition for higher order squeezing:

The normalized condition of higher order squeezing(Hong Mandel type) is derived as

$$S_{HM}(n) = \frac{\langle (\Delta X)^n - \left(\frac{1}{2}\right)_{\frac{n}{2}}}{\left(\frac{1}{2}\right)_{\frac{n}{2}}} < 0 \tag{3}$$

where *nth* order moment for usual quadrature variable ($X = \frac{1}{\sqrt{2}}(a + a^{\dagger})$) is given as

$$\langle (\Delta X)^n \rangle = \sum_{r=0}^n \sum_{i=0}^{\frac{r}{2}} \sum_{k=0}^{r-2i} (-1)^r \frac{1}{2^{\frac{n}{2}}} t_{2i} \, {}^{r-2i} C_k \, {}^n C_r \, {}^r C_{2i} \langle a^{\dagger} + a \rangle^{n-r} \langle a^{\dagger k} a^{r-2i-k} \rangle. \tag{4}$$

To be precise, to study the possibility of HOS for an arbitrary quantum state $|\psi\rangle$ we just need to calculate $\langle a^{\dagger} + a \rangle$ and $\langle a^{\dagger k} a^{r-2i-k} \rangle$. Calculation of these expectation values are simple as that can be done using the

analytical tools like MAPLE and MATHEMATICA.

Criterion for higher order antibunching:

The criterion of HOA is expressed as

$$d(l) = \left\langle a^{\dagger l+1} a^{l+1} \right\rangle - \left\langle a^{\dagger} a \right\rangle^{l+1} < 0.$$
(5)

Here we can note that d(l) = 0 and d(l) > 0 corresponds to higher order coherence and higher order bunching (many photon bunching) respectively.

Higher order subpoissonian photon statistics:

The condition of higher order subpoissonian photon statistics derived as

$$d_h(n-1) = \sum_{r=0}^n \sum_{k=1}^r S_2(r,k) \, {}^n C_r(-1)^r \langle a^{\dagger k} a^k \rangle \langle a^{\dagger} a \rangle^{n-r} - \sum_{r=0}^n \sum_{k=1}^r S_2(r,k) \, {}^n C_r(-1)^r \langle a^{\dagger} a \rangle^{k+n-r} < 0.$$
(6)

The negativity of $d_h(n-1)$ will mean (n-1)th order subpoissonian photon statistics.

Quantum phase fluctuation and nonclassicality:

The U parameter as a measure of phase fluctuation is defined as:

$$U\left(\theta, |\alpha|^{2}\right) = (\Delta N)^{2} \left[(\Delta S)^{2} + (\Delta C)^{2} \right] / \left[\langle S \rangle^{2} + \langle C \rangle^{2} \right]$$

$$\tag{7}$$

where, θ is the phase of the input coherent state $|\alpha\rangle$, and $|\alpha|^2$ is the mean number of photon prior to the interaction. The value of U in coherent (poissonian) state is $\frac{1}{2}$ our requirement of strong nonclassicality reduces to $d_u = U - \frac{1}{2} < 0$ Further simplification of the criterion is possible in BP formalism and consequently our requirement for strong nonclassicality is

$$d_u = [\langle a^{\dagger 2}a^2 \rangle + \langle a^{\dagger}a \rangle - \langle a^{\dagger}a \rangle^2] [\frac{\langle a^{\dagger}a \rangle - \langle a^{\dagger} \rangle \langle a \rangle + \frac{1}{2}}{\langle a^{\dagger} \rangle \langle a \rangle}] - \frac{1}{2} < 0.$$
(8)

Now in light of this criterion we have studied the nonclassical behavior of intermediate states.

Quantitative measure of the single photon generation probability:

Implementation of infinitely secured quantum cryptography requires single photon sources (SPSs). Photon statistics of a good SPS is expected to be nonclassical and it is shown that the potential SPSs satisfy criterion of HOA. Here we have introduced a parameter to compare the single photon generation capacity of two quantum states. The parameter, which is the ratio of the number of pulses containing just one photon to the number of pulses containing more than one, is defined as

$$\eta_k = \frac{P_{1,k}}{1 - (P_{0,k} + P_{1,k})} = \frac{|c_{1,k}|^2}{1 - (|c_{0,k}|^2 + |c_{1,k}|^2)}$$
(9)

where $P_{i,k}$ is the probability of getting *i* photon in state *k*. The greater is η_k the better is the single photon source (SPS) and η_k can be maximised with respect to different parameters that define the particular quantum state.

APPENDIX-C

Higher order nonclassicalities in Binomial state:

In the present work we have studied possibilities of observing higher order nonclassicalities in different intermediate states. Table 1 provides a glimpse of observations. Here we are providing some analytic expressions related to criteria of nonclassicality for BS. It is beyond scope of the present synopsis to describe the mathematical derivations of these expressions and the analytic expressions corresponding to other intermediate states studied. They may be obtained in our recent reports mentioned earlier and the detail calculation will also be reported in the proposed thesis.

Binomial state is defined as

$$|p,M\rangle = \sum_{n'=0}^{M} B_{n'}^{M} |n'\rangle = \sum_{n'=0}^{M} \sqrt{{}^{M}C_{n'}p^{n'}(1-p)^{M-n'}} |n'\rangle \quad 0 \le p \le 1.$$
(10)

This state³ is called intermediate state as it reduces to number state in the limit $p \to 0$ and $p \to 1$ (as $|0, M\rangle = 0$ and $|1, M\rangle = |M\rangle$) and in the limit of $M \to \infty$, $p \to 1$, where α is a real constant, it reduces to a coherent state with real amplitude.

Using the above definition of BS and criteria mentioned in Appendix-B, we obtain

$$d(l)_{BS} = \frac{M! p^{l+1}}{(M-l-1)!} - (Mp)^{l+1},$$
(11)

$$d_h(n-1)_{BS} = \sum_{r=0}^n \sum_{k=1}^r \left[S_2(r,k)^n C_r(-1)^r (Mp)^{n-r} \left(\frac{M!p^k}{(M-k)!} - (Mp)^k \right) \right],$$
(12)

$$d_{U(BS)} = \left[\frac{Mp(1-p)}{(\sum_{n=0}^{M-1} B_n^{M-1} B_n^M)^2} \quad \left[\frac{1}{2Mp} + 1 - (\sum_{n=0}^{M-1} B_n^{M-1} B_n^M)^2\right] - \frac{1}{2}\right]$$
(13)

 and

$$S_{HM}(n)_{BS} = \frac{1}{\left(\frac{1}{2}\right)_{\frac{n}{2}}} \left[\sum_{r=0}^{n} \sum_{i=0}^{\frac{r}{2}} \sum_{k=0}^{r-2i} (-1)^{r} \frac{1}{2^{\frac{n}{2}}} t_{2i}^{r-2i} C_{k}^{n} C_{r}^{r} C_{2i} \left[2(Mp)^{1/2} \sum_{n'=0}^{M-1} B_{n'}^{M} B_{n'}^{M-1} \right]^{n-r} \left[\frac{M!^{2} p^{r-2i}}{(M-k)!(M-r+2i+k)!} \right]^{1/2} \sum_{n'=0}^{M-Max[k,r-2i-k]} B_{n'}^{M-k} B_{n'}^{M-r+2i+k} - 1$$

$$(14)$$

$$\eta_k = \frac{{}^{M}C_1\left(\frac{\gamma}{M}\right)\left(1-\frac{\gamma}{M}\right)^{M-1}}{1-\left[\left(1-\frac{\gamma}{M}\right)^{M-1}\left({}^{M}C_0\left(1-\frac{\gamma}{M}\right)+{}^{M}C_1\left(\frac{\gamma}{M}\right)\right)\right]}$$
(15)

Expressions shown here are graphically represented in Fig. 1 and Fig. 2. From the Fig. 1a it is clear that the BS shows HOA and HOSPS simultaneously. This result is not of much interest as we have shown in chapter 2 that the BS is always higher order antibunched and as every higher order nonclassical state is expected to show HOSPS, independent of whether they show HOA and Hong-Mandel squeezing or not. The result of real physical relevance appears when we look at Fig. 1b which shows that the BS does not show Hong Mandel squeezing for all values of p. For example 4th order Hong Mandel squeezing vanishes for M = 50 and $p \ge 0.8607$. In this range the state is still nonclassical and shows HOA and HOSPS but does not show Hong Madel Squeezing. Consequently we can conclude that the HOA and Hong Mandel squeezing are two independent processes which may or may not appear together. Further it can be observed that for

³The state is named as binomial state because the photon number distribution associated with this state $(i.e. |B_n^M|^2)$ is simply a binomial distribution.

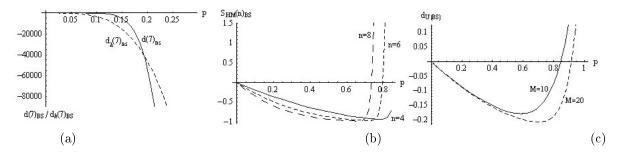


Figure 1: (a) HOA and HOSPS is seen in binomial state, (b) HOS (of different order) in binomial state, (c) Reduction of U in binomial state.

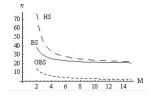


Figure 2: Comparison of single photon generation probability of Binomial State with Generalized Binomial state (GBS) and Hyper Geometric State (HS).

the same photon number (M), the region of nonclassicality decreases with the increase in order of Hong Mandel squeezing. To be precise, when M = 50 then $S_{HM}(4)_{BS}$ is negative till p = 0.8607, but $S_{HM}(6)_{BS}$ is negative till p = 0.7943 and $S_{HM}(8)_{BS}$ is negative till p = 0.7343. In binomial state, parameter η_k is observed to decrease with increasing values of γ or M. In a similar way we can obtain analytic expressions for other intermediate states too. Comparison of single photon generation probability of binomial state with generalized binomial state (GBS) and hyper geometric state (HS) is shown in Fig 2 keeping same value of $\gamma = 0.1$ and other common parameters. In case of GBS $\beta = 10$ is chosen. The comparison shows that HS is a better candidate of SPS.