SOME INVESTIGATIONS IN FUZZY AUTOMATA

Synopsis of the Thesis submitted in fulfillment of the requirements for the Degree of

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Synopsis

Mathematical models in classical computation, automata have been an important area in theoretical computer science [1]. It started from a seminal paper of Kleene [2], and within a few years developed into a rich mathematical research topic. From the very beginning finite automata constituted a core of computer science. Part of the reason is that they capture something very fundamental as is witnessed by a numerous different characterizations of the family of rational languages, i.e. languages defined by finite automata [1]. In fact, the interrelation of finite automata and their applications in computer science is a splendid example of a really fruitful connection of theory and practice and these will accept regular language [3]. Finite automata played a crucial role in the theory of programming languages, compiler constructions, switching circuit designing, computer controller, neuron net, text editor and lexical analyzer [4].

The Myhill–Nerode theory [5] is a branch of the algebraic theory of languages and automata in which formal languages and deterministic automata are studied through right invariant equivalence classes (also studied through right congruences and congruences on a free monoid). It provides necessary and sufficient conditions for a language to be regular, in terms of right invariant equivalence classes. However, the Myhill–Nerode theory not only deals with this theorem, but it also considers many other important topics. In particular, right invariant equivalence classes have shown oneself to be very useful in the proof of existence and construction of the minimal deterministic automaton recognizing a given language, as well as in minimization of deterministic automata [5].

Among the various classical changes in science and mathematics in the previous century, one such change concerns the concepts of uncertainty. According to the traditional view, science should strive for certainty in all its manifestations hence, uncertainty (vagueness) is regarded as unscientific. According to modern view, uncertainty is considered essential to science; it is not only an unavoidable plague, but it has, in fact, a great utility. Zadeh [6] in 1965 introduced the concept of "*Fuzzy Sets*" to describe vagueness mathematically in its abstract form and tried to solve such problems by giving a grade of membership to each member of a given set. This in fact laid the foundations of fuzzy set theory. He stated that the "*membership*" in a fuzzy set is not a matter of affirmation or denial, but rather a matter of degree. Over the last forty years, his proposal has gained recognition as an important point in the evolution of modern concept of imprecision and uncertainty and his innovation represent a paradigm shift from the classical sets or the crisp sets to "*Fuzzy Sets*". The fuzzy set theory has a wider scope of applicability than classical set theory in solving various problems [7-11]. Fuzzy set theory in the last four decades has developed along two lines.

- as a formal theory formulated by generalizing the original ideas and concepts in classical mathematical areas.
- as a very powerful modelling language, that copes with a large fraction of uncertainties of real life situations.

Zadeh has defined a fuzzy set as a generalization of the characteristic function of a subset. A fuzzy set *A* in *U*, the universe of discourse under discussion is identified by a membership function $\mu_A : U \to [0,1]$ defined such that for any element *x* in *U*, $\mu_A(x)$ is a real number in the closed interval [0, 1] indicating the degree of membership of *x* in *A* [6]. The nearer the value of an element to unity, the higher the grade of its membership. Intuitively, a fuzzy set *A* has an unclear, ill-defined boundary so that an element *x* is not necessarily either '*in A*' with $\mu_A(x) = 1$ or '*not in A*' with $\mu_A(x) = 0$; rather, *x* may have partial membership in *A* with $0 < \mu_A(x) < 1$. The membership function μ_A can be viewed as an arithmetization which reflects the ambiguity of set *A*. Since this single number does not tell us the uncertainty/impreciseness completely, we need to generalize the membership function. As a result we come across many extensions of fuzzy sets and among them we have considered intuitionistic fuzzy set (IFS) [12], interval-valued fuzzy set (IVFS) [13-15], and vague set (VS) [16]. IFSs, IVFSs, and VSs are three intuitively straightforward extensions of Zadeh's fuzzy sets that were considered independently to improve the preciseness of the belongingness of an element to a set.

In 1983, Atanassov [12] proposed a generalization of fuzzy set known as intuitionistic fuzzy set. He introduced a new component degree of non-membership in addition to the degree of membership in fuzzy sets with the requirement that their sum be less than or equal to unity. The complement of the two degrees to one is regarded as a degree of uncertainty.

IFS theory basically defies the claim that an element *x* ''belongs'' to a given degree (say μ) to a fuzzy set *A*, naturally follows that *x* should ''not belong'' to *A* to the extent $1-\mu$, an assertion implicit in the concept of a fuzzy set. On the contrary, IFSs assign to each element of the universe both a degree of membership μ and degree of non-membership γ such that $0 \le \mu + \gamma \le 1$, thus relaxing the enforced duality $\gamma = 1 - \mu$ from fuzzy set theory. Obviously, when $\mu + \gamma = 1$ for all elements of the universe, the traditional fuzzy set concept is recovered. IFSs owe their name to the fact that this latter identity is weakened into an inequality, in other words: a denial of the law of the excluded middle occurs, one of the main ideas of intuitionism. Since then a great number of theoretical and practical results appeared in the area of intuitionistic fuzzy sets [17-24].

IVFS theory emerged from the observation that in many cases, no objective procedure is available to select the crisp membership degrees of elements in a fuzzy set. It was suggested to lighten that problem by allowing an interval $[\mu_1, \mu_2]$ to which the actual membership degree is assumed to belong. A related approach, second-order fuzzy set theory, also introduced by Zadeh, goes one step further by allowing the membership degrees themselves to be fuzzy sets in the unit interval; this extension is not considered in this thesis. The theory and applications (some) of IVFSs has been developed in many areas so far as given in [25-32].

Vague sets [16] came into picture due to the concept of membership value of an element of fuzzy set. It says that the single entry combines the evidence for $x \in X$ and the evidence against $x \in X$ without indicating how much of each is there and thus tells us nothing about its accuracy. To overcome this difficulty, vague set allows the membership value in a continuous interval of real numbers in the range [0, 1]. This subinterval keeps the track of both the favouring evidence and the opposing evidence respectively called the truth membership function $t_A(x)$ and false membership function $f_A(x)$ to record the lower bounds on $\mu_A(x)$. These lower bounds are used to create a subinterval of [0, 1], namely $[t_A(x), 1 - f_A(x)]$, to generalize the $\mu_A(x)$ of fuzzy sets. The lower bound and upper bound of this subinterval are $t_A(x)$ and $1 - f_A(x)$, respectively.

All three approaches, IFS, IVFS, and VS theory, have the virtue of complementing fuzzy sets, with ability to model uncertainty as well. IVFSs reflect this uncertainty by the length of the interval membership degree $[\mu_1, \mu_2]$, while in IFS theory for every membership degree (μ, γ), the value $\pi = 1 - \mu - \gamma$ denotes a measure of non-determinacy (or undecidedness), and in VS theory, the subinterval $[t_A(x), 1 - f_A(x)]$ of [0, 1] generalize the membership value $\mu_A(x)$ of fuzzy sets. Each approach has given rise to an extensive literature covering their respective applications, but surprisingly very few people seem to be aware of their equivalence, stated in [33-34]. Indeed, take any IVFS *A* in a universe *X*, and assume that the membership degree of *x* in *A* is given as the interval $[\mu_1, \mu_2]$. Obviously, $\mu_1 + 1 - \mu_2 \leq 1$, so by defining $\mu = \mu_1$ and $\gamma = 1 - \mu_2$ we obtain a valid membership and non-membership degree for *x* in an IFS A'. Conversely, starting from any IFS A' we may associate to it an IVFS *A* by assigning, for each element *x*, the membership degree of *x* in *A* equal to the interval $[\mu, 1-\gamma]$ with again (μ, γ) the pair of membership/non-membership degrees of *x* in A' [35-37]. Also, the membership degree and non-membership degree of an element *x* in an IFS can be written as a truth membership value and false membership value of an element *y* respectively in a VS [38]. As a consequence, a considerable work has been duplicated by adepts of either theory, or worse, is known to one group and ignored by the other. Therefore, regardless of the meaning (semantics) that one likes his or her preferred approach to convey, it is worthwhile to develop the underlying theory in a framework as abstract and general as possible [39].

Since the introduction of fuzzy sets as a method for representing uncertainty, this idea has been applied to a wide range of scientific areas. One such area is automata theory and language theory first introduced by Wee in [40]. There is a deep reason to study fuzzy automata: several languages are fuzzy by nature (e.g. the language containing words in which many letters ‘‘a’’ occur) [41]. The basic idea in the formulation of a fuzzy automaton is that, unlike the classical case, the fuzzy automaton can switch from one state to another one to a certain (truth) degree. Thus, researching fuzzy automaton with ability of processing fuzzy processes is needed. It will process continuous inputs and outputs [42-44]. Even when a system input at a time is missing, the system can work accurately. Obviously, this system is robust and suitable to pattern recognition [45], neural networks [46], lexical analysis [47], clustering, inference and fuzzy control (see [48-50] for applications). Analogously as in the theory of classical automata there are several definitions of a fuzzy automaton (see [51-59] for further references).

Fuzzy automata are the machines accepting fuzzy regular language [47]. This language is a feature of fuzzy language [60] and is described by fuzzy regular expressions [61]. A fuzzy language is generated by a fuzzy grammar, the natural generalization of formal grammar which is introduced to reduce the gap between formal language and natural language (another way of reducing their gap is by introducing randomness [62-63]). Fuzzy languages and fuzzy grammars were introduced by Lee and Zadeh [60]. Fuzzy grammars have been found to be useful in the analysis of X-rays [64]. A fuzzy language $\tilde{L}$, in the set of finite alphabet Σ is a class of strings $w \in \Sigma^*$ along with a grade of membership function $\mu_{\tilde{L}}(w)$. This membership function $\mu_{\tilde{L}}(w), w \in \Sigma^*$, assigns to each string a grade of membership in [0, 1]. This single value combines the evidence for $w \in \Sigma^*$ and the evidence against $w \in \Sigma^*$, without indicating how much of each is there. This single number tells us nothing about its accuracy. To overcome this difficulty, we need to generalize the grade of membership function $\mu_{\tilde{L}}(w)$ of fuzzy languages. The literature on fuzzy language is given in [65-73].

Fuzzy finite automata are used to design complex system. Finding a minimum representation of fuzzy finite automata is critical in such design. The idea of minimizing fuzzy finite automaton was exploited in the papers by Peeva [74], Malik et. al [75], and in Lee [76]. In above papers, fuzzy automaton was reduced by computing and merging indistinguishable states. However, the term minimization used in the mentioned papers does not mean the usual construction of the minimal one in the set of all fuzzy automata recognizing a given fuzzy language. But it is only the procedure of computing and merging indistinguishable states that may not result in a minimal fuzzy automaton.

In this thesis, the generalization of membership function of fuzzy language has been achieved using the concept of extended fuzzy sets namely, intuitionistic fuzzy sets, interval-valued fuzzy sets, and vague sets (other extensions of fuzzy sets are not discussed in this thesis). These respectively results in intuitionistic fuzzy language, interval-valued fuzzy language, and vague language. One can consider these as extended fuzzy languages. The algebraic properties of each language have been studied and are recognized respectively by finite automata with intuitionistic fuzzy transitions, interval-valued fuzzy transitions and vague transitions. Also, the proposed languages are accepted by intuitionistic fuzzy (final) states automata, interval-valued fuzzy (final) states automata, and vague (final) states automata respectively. These extended fuzzy languages are represented by their respective regular expressions. Myhill-Nerode theorem has been extended for each of them and an algorithm is proposed to minimize deterministic finite automata with intuitionistic fuzzy (final) states, interval-valued fuzzy (final) states, and vague (final) states. Finally, the relation between their membership values is obtained.

Approximate string matching [77] is a recurrent problem in many branches of computer science, with applications to text searching, computational biology, pattern recognition, signal processing, etc. It is an offspring of the much simpler exact string matching problem. Use of the term approximate merely emphasizes the fact that a perfect match may not be achievable and imperfections such as missing and extraneous symbols have to be considered between two strings to be compared. Many fuzzy automaton models have been introduced in the past for imperfect string matching [78-80]. This thesis proposes extended fuzzy automaton models, such as intuitionistic fuzzy automaton model, interval-valued fuzzy automaton model, and vague finite automaton model for approximate string matching using the notion of intuitionistic fuzzy sets, interval-valued fuzzy sets, and vague sets respectively. An algorithm has been proposed for each of the case. The returned value is a percentage that tells how close and far the strings that have been compared. We can convert one model into other and thereby generalize the fuzzy automaton process of approximate string matching. The proposed extended fuzzy automaton methods may be applied in text searching, computational biology, pattern recognition, signal processing, etc.

Main Contribution of the Thesis

The thesis is organized chapter wise as follows:

Chapter 1: This chapter is introductory and sets up the background for the problems taken up in the thesis. It overviews finite automata, types of finite automata and relation between them, regular language, and minimization of finite automata by stating Myhill-Nerode theorem. The concept of fuzzy sets has been outlined with some basic operations. Three extensions of fuzzy sets such as intuitionistic fuzzy set, interval-valued fuzzy set and vague set and some of their basic operations including a relation between their membership values are given. Fuzzy grammar and fuzzy language and some of their algebraic operations are also discussed. The concept of fuzzy automaton, the model accepting fuzzy language and its minimization is briefed. Chapter ends with the discussion of approximate string matching in the framework of extended fuzzy automaton.

Chapter 2: This chapter proposes intuitionistic fuzzy regular language, an extension of fuzzy regular language using the concept of intuitionistic fuzzy sets. Basic operations such as union, intersection, complement, concatenation, and star operations on these constructed languages are given. Finite automaton (deterministic and nondeterministic) with intuitionistic fuzzy transitions and intuitionistic fuzzy (final) states has been constructed to recognize the proposed languages. It is observed that, the finite automaton (deterministic and nondeterministic) with intuitionistic fuzzy (final) states is more suitable to recognize intuitionistic fuzzy regular language than the finite automaton (deterministic and nondeterministic) with intuitionistic fuzzy transitions. An attempt has been made to express this language through intuitionistic fuzzy regular expressions. Myhill-Nerode theorem is studied in the framework of intuitionistic fuzzy regular language and an algorithm is given to minimize the redundant states of a deterministic finite automaton with intuitionistic fuzzy (final) states.

Chapter 3: This chapter introduces interval-valued fuzzy regular language, another extension of fuzzy regular language using the notion of interval-valued fuzzy sets. Some of the basic algebraic operations namely union, intersection, complement, concatenation, and star operations of the introduced language are given. Finite automaton (deterministic and nondeterministic) with interval-valued fuzzy transitions and interval-valued fuzzy (final) states automata are proposed to discuss the recognition of interval-valued fuzzy regular language through some theorems. The description of interval-valued fuzzy regular language is attained through interval-valued fuzzy regular expression. Finally, minimization of deterministic finite automaton with interval-valued fuzzy (final) states is achieved through Myhill-Nerode theorem for interval-valued fuzzy regular language and an algorithm is given for the same.

Chapter 4: Vague regular language is proposed in this chapter using the concept vague sets, one more generalization of fuzzy sets. Some algebraic operations on these languages are given. Chapter also proposes finite automata (deterministic and nondeterministic) with vague transitions and finite automata (deterministic and nondeterministic) with vague (final) states to study vague regular language through some theorems. It gives vague regular expressions for useful representation of strings of vague regular language in an algebraic fashion. Furthermore, extended Myhill-Nerode theorem is discussed for vague regular language and an algorithm to minimize deterministic finite automaton with vague (final) states is given. While concluding the chapter, a relation between the membership values of the above proposed extended fuzzy languages (in sequence; intuitionistic fuzzy language, interval-valued fuzzy language, and vague language) has been given.

Chapter 5: This chapter discusses the application of proposed extended fuzzy automata given in chapter 2, chapter 3 and chapter 4. It presents the approximate string matching using intuitionistic fuzzy automaton, interval-valued fuzzy automaton, and vague finite automaton accepting respectively, intuitionistic fuzzy regular language, interval-valued fuzzy regular language, and vague regular language. These methods, models the possible edit operations such as insertion, substitution and deletion needed to transform an observed string (input string) into a pattern string (target string) by providing a suitable membership value between them. These will generalize conventional fuzzy model for approximate string matching. The selection of appropriate extension of fuzzy automaton models with their membership values for the transitions leads to improve the system performance for a particular application. A relation between the membership values between the pair of strings of extended fuzzy automata is obtained. Chapter ends by giving an algorithm for approximate string matching in each of three cases.

Scope of future work:

Further work can be done in the following areas:

1. Algebraic properties of intuitionistic fuzzy sets, interval-valued fuzzy sets, and vague sets can be further studied with intuitionistic fuzzy language, interval-valued fuzzy language, and vague language respectively.
2. Fuzzy language has been applied so far in the analysis of X-rays, in digital circuit design and one can think of studying these by applying proposed extended fuzzy languages.
3. These extended fuzzy languages can be applied in the study of lexical analysis.
4. Category theory may be studied with the help of above proposed languages.

RESEARCH PAPERS CONTRIBUTION IN THE THESIS

INTERNATIONAL REFEREED JOURNALS:

1. Choubey A. and Ravi K. M., “Intuitionistic Fuzzy Automata and Intuitionistic Fuzzy Regular Expressions,” J. Appl. Math. & Informatics, vol. 27, no. 1-2, pp. 409-417, 2009.
2. Ravi K. M. and Choubey A., “Interval-Valued Fuzzy Regular Language,” J. Appl. Math. & Informatics, vol. 28, no. 3-4, pp. 639-649, 2010.
3. Choubey A. and Ravi K. M., “Minimization of DFA-with Intuitionistic fuzzy (final) stats and Vague (final) states,” communicated to The Iranian Journal of Fuzzy Systems (received and responded to referee’s report).
4. Ravi K. M. and Choubey A., “Approximate string matching by Intuitionistic Fuzzy Automaton,” communicated to the European Journal of Fuzzy Mathematics.
5. Ravi K. M. and Choubey A., “Approximate string matching by Extended Fuzzy Automaton,” communicated to Belgium Mathematical Society.

NATIONAL REFEREED JOURNALS:

1. Choubey A. and Ravi K. M., “Vague Regular Language,” Advances in Fuzzy Mathematics, ISSN 0973-533X, vol. 4, pp. 147-165, 2009.

INTERNATIONAL/NATIONAL CONFERENCES:

1. Ravi K. M. and Choubey A., “Intuitionistic Fuzzy Regular Language,” Proceedings of International Conference on Modelling and Simulation, CITICOMS, Aug 2007, Coimbatore, India, ISBN No. 81-8424-218-2, pp. 659-664, 2007.
2. Ravi K. M. and Choubey A., “Myhill-Nerode theorem for Interval-valued Fuzzy Regular Language,” International Conference on Methods and Models in Science and Technology (ICM2ST-10), Chandigarh, Dec 2010, India. Published in Conference Proceedings of American Institute of Physics, Nov 6, 2010, vol. 1324, no. 1, pp. 30-34 (Indexed in Scopus).

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