SOME STABILITY PROBLEMS OF NON-NEWTONIAN FLUIDS

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By

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INTRODUCTION

The science which deals with the properties of fluids in motion is called Fluid Dynamics. Fluid dynamics has a wide range of applications, including calculating forces and moments on aircraft, predicting weather patterns, the extraction of oils from underground reservoirs; influence of wind upon building structures, etc. Some of its principles are even used in traffic engineering, where traffic is treated as a continuous fluid.

The subject dealing with the motion of electrically conducting fluids in the presence of magnetic field is termed as magnetohydrodynamics (MHD) or hydromagnetics. Examples of such fluids include plasmas, liquid metals and salt water or electrolytes. The field of MHD was initiated by Hannes Alfvén [1] for which he received the Nobel Prize in Physics in 1970. Ferrohydrodynamics (FHD) deals with the mechanics of fluid motions influenced by strong forces of magnetic polarization. In general, strong thermo mechanical coupling exists when the induced polarization is both temperature and field dependent. In MHD the body force acting on the fluid is the Lorentz force that arises when electric current flows at an angle to the direction of an impressed magnetic field. However, in FHD, usually no electric current is flowing in the fluid.

Every system in nature is subjected to small perturbations. The system is given small disturbances (perturbations) and sees the reaction of these perturbations. If the disturbances gradually die down, the system is said to be stable. If the perturbations grow with time i.e. the system never reverts to its initial position, it is said to unstable. If the system neither departs from its disturbed state nor tends to return to its initial position, the system is said to be in neutral equilibrium. Further if at the onset of instability, there is an oscillatory motion with growing amplitude; the instability is termed as overstability. Instability of the system cannot be termed as stable unless it is stable with respect to every possible disturbance to which it can be subjected.

Thermal instability theory (or Bénard convection) has attracted considerable interest and has been recognized as a problem of fundamental importance in many areas of fluid dynamics. The earliest experiments to demonstrate the onset of thermal instability in fluids are attributed to Bénard [2]. In the majority of situations, if a layer of fluid is heated from below, the fluid in the lower part of the layer expands as it becomes hotter. Thus, on the account of thermal

convection, the fluid at the bottom will be lighter than the fluid at the top. This is a top-heavy arrangement, which is potentially unstable and when the temperature gradient or layer depth is sufficiently large to overcome the effect of gravity, the fluid rises and a pattern of cellular motion may be seen. This is called Bénard convection (Bénard [2]). It is of historical interest to point out that the tessellated structure was previously observed by Rumford [3] and Thomson [4]. Bénard carried out his experiment on a very thin layer of non-volatile liquid (1 mm in depth) placed on a carefully leveled metallic plate maintained at a constant temperature. The upper surface of layer was kept in contact with the free air. It was found that the layer resolved itself into a number of cells, known as Bénard cells.

In the standard Bénard problem, the instability is driven by a density difference which is caused by a temperature difference between the upper and lower planes bounding the fluid. If the fluid layer additionally has salt dissolved in it, then there are potentially two destabilizing sources for the density difference i.e. the temperature field and the salt field. When the simultaneous presence of two or more components with different diffusivities is considered, the phenomenon of convection which arises is called thermosolutal or double-diffusive convection. The problem of thermosolutal convection in a layer of fluid heated from above has been studied by Stern [5] and Aggarwal and Prakash [6]. Veronis [7] studied Stern's problem when the fluid layer is heated and soluted from below. The same problem has been considered by Nield [8] when the fluid layer is heated from below and soluted from above.

Thermal stability and thermosolutal stability of hydrodynamic and hydromagnetic systems of various non-Newtonian fluids viz. Walters', Rivlin-Ericksen, couple-stress, ferromagnetic and micropolar fluids have been considered. A good review of thermal stabilities has been given in the celebrated monograph by Chandrasekhar [9] and Drazin & Reid [10]. The linearized stability theory and normal mode analysis are used to determine the critical values of Rayleigh number, demarcating a region of stability from instability. Linear theory gives the conditions under which hydrodynamic systems are definitely unstable. Normal mode analysis is quite general and it gives complete information about instability, including the rate of growth of any unstable perturbation.

In the study of problems of thermal convection, it is a frequent practice to simplify the basic equations by introducing certain approximations which are attributed to Boussinesq [11]. The Boussinesq approximation can best be summarized by two statements: (i) The

fluctuations in density which appear with the advent of motion result principally from thermal (as opposed to pressure) effects. (ii) In the equations for the rate of change of momentum and mass, density variations may be neglected except when they are coupled to the gravitational acceleration in the buoyancy force. The formulation and derivation of the basic equations of a layer of a fluid heated from below in porous medium, using Boussinesq approximation, has been given in a treatise by Joseph [12].

The Rivlin-Ericksen elastico-viscous fluid (Rivlin and Ericksen [13]) and other polymers are used in the manufacture of parts of space crafts, aeroplane parts, tyres, belt conveyers, ropes, cushions, seats, foams, plastics, adhesives, engineering equipments, contact lens etc. Recently, polymers are also used in agriculture, communication appliances and in biomedical applications. Rivlin-Ericksen elastico-viscous fluids form the basis for the manufacture of many such important and useful products. Sharma and Kumar [14] have studied the thermal instability of Rivlin-Ericksen elastico-viscous fluid in hydromagnetics whereas the thermal convection in Rivlin-Ericksen viscoelastic fluid in porous medium in hydromagnetics has been studied by Sharma and Kango [15].

A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as the lubricant. When a fluid film is generated, squeeze-film action is capable of providing considerable protection to the cartilage surface. The shoulder, hip, knee and ankle joints are the loaded-bearing synovial joints of the human body and these joints have a low friction coefficient and negligible wear. Normal synovial fluid is a viscous, non-Newtonian fluid and is generally clear or yellowish. The theory of couple-stress fluid has been formulated by Stokes [16]. The synovial fluid has been modeled as a couple-stress fluid in human joints by Walicki and Walicka [17]. The couple-stress fluid heated from below in presence of magnetic field and rotation has been considered by Sunil et al. [18].

Magnetic fluids are electrically non conducting colloidal suspension of solid ferromagnetic magnetite particles in a non-electrically conducting carrier fluid like water, kerosene, hydrocarbon etc. A typical ferromagnetic fluid contains 10^{23} particles per cubic meter. Ferromagnetic fluids are not found in nature but are artificially synthesized. The importance of ferrofluids was realized soon after the method of formation of ferrofluids, during mid sixties. This is because it had a very large potential application in various fields, however, due to the availability of colloidal magnetic fluids, many other uses of these

fascinating liquids have been identified which are governed with the remote positioning of magnetic field controlling the magnetic fluid. Rosensweig [19] has given an authoritative introduction to the research on magnetic liquids in his monograph, and the study of the effect of magnetization yields interesting information. The theory of convective instability of ferromagnetic fluids began with Finlayson [20] and was interestingly continued by Lalas & Carmi [21], Shliomis [22], Schwab et al. [23], Stiles and Kagan [24], Blennerhassett et al. [25], Venkatasubramanian and Kaloni [26] and Sunil et al. ([27], [28], [29], Aggarwal and Prakash [30]).

Eringen ([31]-[33]) introduced the theory of micropolar fluid, in which the fluid elements can undergo micro-rotation, in addition to translation. These two motions give rise to two independent vectors, namely, the velocity vector and the gyration vector (or micro-rotation velocity vector). The velocity vector comes into play due to translatory motion, while the gyration vector arises due to micro-rotatory motion. Each of these vectors will have three components and hence the model of micropolar fluid will have six degrees of freedom. Micropolar fluid can support couple stresses, in addition to usual force stresses and may possess micro-inertia. The basic difference between micropolar fluids and that of the classical fluids is that they can support couple stresses due to rotatory motion, and the force stresses due to translatory motion. The fluids containing elongated molecules, for example, polymeric fluids, liquid crystals with rigid molecules, magnetic fluids, animal blood, etc may fall into the category of micropolar fluids.

In the literature, the effects of various parameters such as porous medium, magnetic field, suspended particles, rotation, compressibility, Hall currents etc. on the thermal stability of hydrodynamic and hydromagnetic systems is studied. A brief discussion and development of these parameters is given hereunder.

A medium which is a solid body containing pores is called a porous medium. Flow through porous media is also of interest in chemical engineering (adsorption, filtration, flow in packed columns), petroleum engineering, hydrology, soil physics, biophysics and geophysics. Two macroscopic properties of porous media which may be used to describe fluid flow are porosity (ε) and permeability (k_1). The porosity of a porous medium describes how densely the material is packed. The porosity of a porous medium is defined as the fraction of the total volume of the medium that is occupied by the void space. Thus $1-\varepsilon$ is the fraction that is occupied by solid. Porosity is a fraction between 0 and 1, typically ranging from less than 0.01 for solid granite to more than 0.5 for peat and clay. For man made materials such as metallic frames, ε can approach to the value 1. For natural media, ε does not normally exceed 0.6.

The permeability is related to pore-size distribution since the distribution of the sizes of entrances, exits and lengths of the pore walls make up the major resistance to flow. The permeability is the single parameter that reflects the conductance of a given pore structure. The dimensions of the permeability are length squared. In oil industry it is measured in 'darcy' with $1 \text{darcy} = 9.87 \times 10^{-9} \text{ cm}^2$. The permeability and porosity are related since if the porosity is zero the permeability is zero.

Consider a fluid to be electrically conducting and be under the influence of a magnetic field. The electrical conductivity of the fluid and the prevalence of magnetic fields contribute to effects of two kinds. First, by the motion of the electrically conducting fluid across the magnetic lines of force, electric current are generated and the associated magnetic fields contribute to changes in the existing fields; and second, the fact that the fluid elements carrying currents traverse magnetic lines of forces contributes to additional forces acting on the fluid elements. It is thus two-fold interaction between the motions and the fields that is responsible for patterns of behaviour, which are often striking and unexpected. The interaction between the fluid motions and magnetic fields are contained in Maxwell's equations. As a consequence of Maxwell's equations, the equations of hydrodynamics are modified suitably.

Various studies in viscous, viscoelastic, couple-stress fluids with suspended particles (dust particles), have appeared in the literature (Saffman [34], Scanlon and Segel [35], Sharma et al. [36], Palaniswamy and Purushotham [37], Aggarwal [38], Sharma et al. [39], Gupta et al. [40] and Sunil et al. [41]) because of the importance of dusty fluids in a wide range of areas of technical importance such as fluidication, environmental pollution and weather forecasting etc. The influence of suspended particles on viscoelastic flows has a great importance in petroleum industry, pulp and paper technology, purification of crude oil and several geophysical situations. The present study of suspended particles can serve as a theoretical support for experimental investigations e.g. evaluating the influence of impurifications in fluids like couple-stress and ferromagnetic fluids on thermal convection phenomena.

The physical aspect of convection in a rotating fluid layer is the driving force for analysis. As convection in a rotating system is relevant to many geophysical applications and to industrial applications such as semiconductor crystal growing, it is not surprising that there have been many articles dealing with theoretical or experimental analysis of this problem (Chandrasekhar [42], Chandrasekhar and Elbert [43], Veronis ([44], [45]), Roberts [46], Veronis [47], Rossby [48], Roberts and Stewartson [49], Sharma and Rana [50] and Langlois [51]). Thermal convection in a rotating layer of a porous medium saturated by a homogeneous fluid is a subject of practical interest for its applications in engineering. Among the applications in engineering disciplines one can find the food process industry, chemical process industry, solidification and centrifugal casting of metals and rotating machinery. More detailed discussions of applications of thermal convection in porous media and particularly in rotating porous domains are presented by Nield and Bejan [52].

Fluids are divided into two categories. Those which undergo appreciable variations in density and volume under the impressed forces fall under the category of 'compressible fluids'. Then, there are those which undergo no noticeable changes in density and volume during motion. They are termed 'incompressible fluids'. Compressibility is thus a measure of the change in density and consequently, the change in volume of a fluid under the effect of external forces. For compressible fluids, the equations governing the system become quite complicated. Spiegel and Veronis [53] have simplified the set of equations governing the flow of compressible fluids assuming that the depth of the fluid layer is much smaller than the scale height as defined by them and the motions of infinitesimal amplitude are considered. Sharma and Aggarwal [54] and Sharma et al. [55] have studied the effect of compressibility and suspended particles on thermal convection in a Walters' elastico-viscous fluid and couple-stress fluid, respectively.

In the presence of strong electric field, the electric conductivity is found to be affected by the magnetic field. Consequently, the conductivity parallel to the electric field is reduced in the direction normal to both electric and magnetic fields. This phenomenon is known as Hall Effect. Gupta [56] studied the effect of Hall currents on the thermal instability of a horizontal layer of electrically conducting fluid. The effect of Hall currents on thermal instability has also been studied by several authors (Raptis and Ram [57], Sharma and Rani [58], Sunil et al. [59], Sharma and Kumar [60], Gupta and Aggarwal [61]).

The non-Newtonian fluids are of vital importance due to their diverse applicational aspects in modern technology, industries and bio-mechanics. Thus, the analysis of thermal stability and thermosolutal stability of such fluids like Rivlin-Ericksen fluids, couple-stress fluids, ferromagnetic fluids and micropolar fluids are desirable. We have used the two fundamental hypotheses i.e., continuum hypothesis and Newtonian mechanics throughout our study. In the present thesis, the linearized stability theory and normal mode analysis have been used to study the effects of various important parameters like suspended particles, compressibility, rotation, magnetic field, Hall currents, solute gradient, variable gravity, porous medium, micropolar coefficient, coupling parameter, micropolar heat conduction parameter etc. on various stability problems of hydrodynamic and hydromagnetic systems of non-Newtonian fluids.

OBJECTIVE

The main objectives of the work reported in the thesis are:

- To study the thermal and thermosolutal stability of hydrodynamic and hydromagnetic systems.
- To study the effect of various parameters like rotation, magnetic field, suspended particles, compressibility, porous medium, micropolar coefficient, coupling parameter, micropolar heat conduction parameter, variable gravity field, Hall currents etc. on various stability problems of non-Newtonian fluids.
- To find the parameters which are responsible for the oscillatory modes.

CONTRIBUTION

The work embodied in the present thesis is divided into six chapters. The problems and the summary of the work done are presented below chapter wise.

CHAPTER-1

Chapter 1 is introductory. It reviews existing literature relevant to the thesis e.g. hydrodynamics, hydromagnetics, ferrohydrodynamics, stability of hydrodynamic and hydromagnetic systems, methods and scopes determining stability etc. The thermal and thermosolutal stability problems have been described and effects of various parameters like suspended particles, Hall currents, uniform/variable magnetic field, rotation, porous medium, permeability, viscoelasticity etc. on the thermal stability of the hydrodynamic/hydromagnetic systems have also been discussed.

CHAPTER-2

Chapter 2 is divided into two sections namely, A and B. In these two sections, the thermosolutal convection in Rivlin-Ericksen fluid in porous medium is studied. In section A, a layer of Rivlin-Ericksen fluid heated and soluted from below in porous medium is considered in the presence of uniform vertical magnetic field, rotation and suspended particles. It is found that for stationary convection, the stable solute gradient and rotation have stabilizing effect on the system whereas suspended particles have destabilizing effect. The medium permeability has a destabilizing effect in the absence of rotation whereas in the presence of rotation it has a destabilizing/stabilizing effect under certain conditions. The magnetic field has a stabilizing effect in the absence of magnetic field, rotation and stable solute gradient. The presence of magnetic field, rotation and stable solute gradient. The presence of magnetic field, rotation and stable solute gradient. The presence of magnetic field, rotation and stable solute gradient introduces oscillatory modes into the system.

In section B, the thermosolutal convection in Rivlin-Ericksen elastico viscous fluid in porous medium is considered to include the effect of suspended particles in the presence of uniform magnetic field, uniform rotation and variable gravity field. It is found that, for stationary convection, the stable solute gradient and rotation have stabilizing effect on the system whereas suspended particles have destabilizing effect. The medium permeability has a destabilizing effect in the absence of rotation whereas in the presence of rotation it has a destabilizing/stabilizing effect under certain conditions. The magnetic field has a stabilizing effect in the absence of rotation whereas in the presence of rotation it has a stabilizing/destabilizing effect under certain conditions. The principle of exchange of stabilities is satisfied in the absence of magnetic field, rotation and stable solute gradient. The presence of magnetic field, rotation and stable solute gradient introduces oscillatory modes into the system.

CHAPTER-3

Chapter 3 is divided into two sections namely, A and B. In these two sections, the effect of Hall currents on instability problems in ferromagnetic fluids is studied. Section A deals with the theoretical investigation of the effect of Hall currents and suspended particles

on thermal stability of a ferromagnetic fluid heated from below. For a fluid layer between two free boundaries, an exact solution is obtained using a linearized stability theory and normal mode analysis. A dispersion relation governing the effects of suspended particles and Hall currents is derived. For the case of stationary convection, it is found that the magnetic field has a stabilizing effect on the system, as such its effect is to postpone the onset of thermal instability whereas the suspended particles and Hall currents are found to hasten the onset of thermal instability, as such they have destabilizing effect on the system. The effects of various parameters on the thermal stability are depicted graphically also. The critical Rayleigh numbers and wave numbers of the associated disturbances for the onset of instability as stationary convection are obtained and the behaviour of various parameters on critical thermal Rayleigh numbers has been depicted graphically. The principle of exchange of stabilities is not valid for the problem under consideration whereas in the absence of Hall currents (hence magnetic field), it is valid under certain conditions.

In section B, problem of section A has been extended to include the effect of compressibility as an additional parameter. For the case of stationary convection, it is found that the compressibility has a stabilizing effect on the system.

CHAPTER-4

Chapter 4 is divided into two sections. In these two sections, the effect of Hall currents on ferromagnetic fluids in porous medium is considered. First section deals with the theoretical investigation of the effect of Hall currents on the thermal stability of a ferromagnetic fluid heated and soluted from below in porous medium. For the case of stationary convection, it is found that the magnetic field and stable solute gradient have a stabilizing effect on the system, as such its effect is to postpone the onset of thermal instability whereas Hall currents are found to hasten the onset of thermal instability, as such it has destabilizing effect on the system. The medium permeability hastens the onset of convection for certain wave numbers. The effects of various parameters on the thermal stability are depicted graphically also. The critical Rayleigh numbers and wave numbers of the associated disturbances for the onset of instability as stationary convection are obtained and the behaviour of various parameters on critical thermal Rayleigh numbers has been depicted graphically. The principle of exchange of stabilities is not valid for the problem under consideration whereas in the absence of stable solute gradient and Hall currents (hence magnetic field), it is valid under certain conditions.

Second section deals with the theoretical investigation of the effect of Hall currents on the thermal stability of a ferromagnetic fluid heated from below in porous medium in the presence of horizontal magnetic field. For the case of stationary convection, it is found that the magnetic field has a stabilizing effect on the system, as such its effect is to postpone the onset of thermal instability whereas Hall currents are found to hasten the onset of thermal instability, as such it has destabilizing effect on the system. The medium permeability hastens the onset of convection for certain range of wave numbers. The effects of various parameters on the thermal stability are depicted graphically. The principle of exchange of stabilities is not valid for the problem under consideration whereas in the absence of stable solute gradient and Hall currents (hence magnetic field), it is valid under certain conditions.

CHAPTER-5

Chapter 5 is divided into two sections. In these two sections, thermal stability of couple-stress fluids is studied. In first section, the thermal stability of a couple-stress fluid in the presence of magnetic field and rotation is considered. For stationary convection, it is found that suspended particles have destabilizing effect whereas rotation has stabilizing effect always. The magnetic field and couple-stresses have a stabilizing effect under certain conditions. In the absence of rotation, couple-stresses and magnetic field have stabilizing effect on the system. It is found that principle of exchange of stabilities is satisfied in the absence of magnetic field and rotation.

In second section, the problem of first section has been extended to include the effect of suspended particles and it is observed that suspended particles have destabilizing effect on the system.

CHAPTER-6

In this chapter, thermal instability problem of micropolar fluid layer heated from below in the presence of Hall currents in porous medium has been addressed. For the case of stationary convection, the effect of various parameters like medium permeability, magnetic field, Hall currents, coupling parameter, micropolar coefficient and micropolar heat conduction parameter has been analyzed and results are depicted graphically also. It is found that the medium permeability has destabilizing effect under certain conditions. The micropolar heat conduction parameter, the magnetic field parameter, Hall current parameter and coupling parameter has stabilizing effect under certain conditions. For small wave numbers $(0 \le k \le 0.7)$, micropolar coefficient has destabilizing effect whereas for k > 0.7, it has stabilizing effect.

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- 3. Aggarwal, A. K. and Makhija, S. "Combined effect of magnetic field and rotation on thermal stability of couple-stress fluid heated from below in presence of suspended particles" International Journal of Applied Mechanics and Engineering, vol.16, No. 4, pp. 931-942, 2011 (indexed in INSPEC, JEE, UK, ASME, USA, Zentralblatt MATH, Germany, etc.).
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