

Some Special Classes of Efficient Multiple Bursts Codes

*Synopsis of the Thesis submitted in fulfillment of the requirements for the
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By

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1. INTRODUCTION

Shannon [29] in his seminal paper ‘*A Mathematical Theory of Communication*’ laid down the mathematical foundation of *Theory of Reliable Communication*. In his paper Shannon gave elegant mathematical characterizations of source, channel, channel capacity and measure of information. He also described salient features of reliable communication over unreliable channels. This marked the birth of *Coding Theory*, as a basic tool for transmission of data across noisy channels and recovery of original messages from the corrupted messages.

Coding theory is mathematically an area which, it is widely acclaimed has enriched applications of mathematics in a rather unprecedented way and in turn has also enriched mathematics. With ever expanding dimensions of the present ‘Information Age,’ coding theory, that has advanced well, presents new challenges both in theory and applications with expanding demands. Talking of areas of applications, coding theory has found practical applications in the areas ranging from communication systems to digital data transmission, to modern medical science, to space communication.

Theoretical foundations of Coding theory are traced to Shannon’s famous ‘Fundamental theorem of Information Theory’ [29] that guarantees *existence* of codes that can transmit information at a rate close to the capacity of the channel with a vanishingly small probability of error. The theory of error-correcting codes developed to realize in theory and practice promise made by Shannon’s fundamental theorem.

Work in this area began with highly significant paper of Hamming [18].Hamming introduced the basic concepts of linear parity check, parity-check matrix and a metric. Early work dealing with the performance of codes in terms of their minimum distance was done by Hamming [18], Gilbert [17], Varshamov [35] and others. They derived bounds on number of parity checks for a minimum distance. One of the most important achievements of coding theory came through the work of Bose and Choudhary [2] and Hocquenghen [19] independently, providing thereby a general method of constructing binary codes capable of correcting *multiple random errors*. These codes, known as BCH codes, were shown to be cyclic by Peterson [24]. A large number of important code constructions which have coefficients including those of BCH codes, use the techniques based on the roots of polynomials which have coefficients in a Galois field.

Random and Burst Errors:

Major attention in coding theory has been given to study of linear block codes correcting random errors. A basic and useful criterion for random error correction in terms of Hamming distance is that a code with minimum distance $2t+1$ can correct any t errors. However, it was noticed later that in many communication channels the occurrence of errors is more often in clusters-in adjacent positions rather than their occurrence in a random manner. In many instances of communication, as is common knowledge, errors do not occur independently but are in many ways clustered. This led to the study of burst error-correcting codes, introduced by Fire [16] and Regier [27] and later nicely treated by Peterson and Weldon [25].

Since the development of various burst error correcting and detecting codes, several variants and modifications of the definition of a burst error came depending upon the nature of channels which are in use. It was noted by Chien and Tang [4] that in several communication channels errors occurs in the form of a burst but not in the end digit of the burst. This type of error in literature is known as CT burst. Further, this definition was modified by Dass [5]. There are some codes that have been developed to deal with multiple bursts, random multiple bursts, block-wise bursts etc. Significant work in the field of multiple burst error correction has been done by Wolf [36], Stone [34], Bridwell and Wolf [3], and others. There are many channels which require development of codes dealing with multiple bursts.

Repeated Burst Errors:

It has been noticed that when there are large number of messages, the code words are quite long and within a given length bursts repeat themselves. This led to the study of repeated burst error detecting and correcting codes. The concept of repeated bursts was introduced by Dass and Verma [12]. Linear codes detecting and correcting repeated burst errors have been studied by them. In their paper, Beraradi, Dass and Verma [1] obtained lower and upper bounds on the number of parity-check digits required for a linear code that is capable of detecting 2-repeated burst errors and also codes capable of detecting and simultaneously correcting such errors have been dealt with by them. In general, Dass and Verma [13] defined m -repeated burst of length b and obtained results regarding the number of parity-check digits for correcting and detecting such type of errors. They also studied codes for correcting 2-repeated burst errors and in general m -repeated bursts of length b or less.

Codes for detecting repeated low-density burst errors of length b or less with weight w or less were also studied by Dass and Verma [14]. In one of their papers Dass and Verma [15] code that is capable of correcting repeated burst errors of length b (fixed). An upper bound derived lower and upper bounds on the number of parity-check digits for codes correcting 2-repeated low-density bursts of length b or less with weight w or less.

Dass, Garg and Zannetti [10] studied another kind of repeated burst errors, i.e., repeated burst errors of length b (fixed). Linear codes capable of detecting 2-repeated bursts of length b (fixed) were studied by Dass and Garg [8]. Dass, Garg and Zannetti [11] obtained lower and upper bound on the number of parity-check digits required for a linear code that is capable of correcting repeated burst errors of length b (fixed). An upper bound on the number of parity-check digits for a code to detect m -repeated burst of length b (fixed) was also derived by them. Dass and Garg [9] also obtained bounds on the number of parity-check digits for a linear code that can detect repeated low-density burst of length b (fixed) with weight w or less.

Studies of Burst Errors with Weight Considerations:

The consideration of weight plays an important role in coding theory. However, mainly only two types of weights given by Hamming [18] and Lee [22] have been used in coding theory. Hamming metric and codes studied on that were binary. But in case of Lee, the code characters are taken from the ring Zq of integers modulo q . Later search for all possible distances was accomplished by Sharma and Kaushik [32], over $GF(q)$. This depends on the partition of $GF(q)$ into subsets with specific structure. However, we in this thesis will confine to Hamming case, that when considered over $GF(q)$, also has weight sensitivity.

Efficiency has been a major consideration in study of error-correcting codes. These considerations require considering, as best as possible, only those errors that can actually occur in the system. Considering in a position possibility of all possible substitution errors, when these are limited to few around brings unnecessary errors for correction. This study of bursts in terms of weight was initiated by Sharma and Dass [30]. Extending their work Krishnamurthy [21] gave some additional combinatorial results regarding the weight of burst errors.

The importance of coding theory lies in applications, and this introduced the study of cyclic codes which are easily transferable through shift registers. Besides being easily

implementable, these were found to be mathematically very sound in structure. Cyclic codes, introduced by Prange [26], are linear block error-correcting codes that have convenient algebraic structures for efficient error detection and correction. In many channels error patterns are not really random, they occur in the form of bursts in segments of a message. So, for correcting bursts of errors we need a more efficient code of higher rate because of fewer constraints. Cyclic codes are very useful for this purpose. Jain [20] studied open-loop burst error detecting and moderate density open-loop burst error detecting cyclic codes.

In general, burst of a given length may not have all digits in it corrupted. In some particular systems such as supercomputer storage system, it is seen that within a burst all the digits are nonzero. Such types of errors are called as solid burst errors. Schillinger [28] developed codes that correct solid burst errors. Shiva and Sheng [33] gave a simple decoding scheme for codes correcting multiple solid bursts in binary case. Studies by Sharma and Dass [31], Das [6] cover notable results on solid burst error correcting codes. Recently Das [7] has undertaken study of 2-repeated solid burst and obtained codes that are capable of detecting and simultaneously correcting these errors.

2. OBJECTIVES OF THE STUDY

Major objective of the study reported in this thesis can perhaps be seen in two different ways:

- (a) From application point of view to get combinatorial results with weight consideration for efficient communication when repeated burst errors occur, which largely occur when messages are of longer lengths. In fact, by considering bursts of all four different types, the study is very broad in its coverage. Our study provides repeated burst correction/detection criteria for different variety of channels that in practice can be used with currently ever widening phenomena of digital communication.
- (b) From theoretical point of view, a major perspective also follows from our study. Reflecting on random and burst errors, as these are, it can be easily seen that burst errors constitute a subclass of random errors. Our study of multiple bursts in message vectors presents a mathematical formulation of a much broader and general class of errors unifying random errors and multiple burst errors. So much so that random error correcting codes can clearly be viewed as a subclass of multiple burst error correcting codes, with every burst of length one in particular. In fact, our study of multiple burst errors is thus a generalization of random errors and burst errors in one stroke.

Specifically speaking of efficiency, if a repeated burst is considered as a single burst or as a bunch of random errors, we will have to use more parity-checks and that will reduce the efficiency of the code.

Areas of problems mathematically handled, in short, can be put as follows:

- Developing the mathematical formulation for weight calculation of various types of repeated burst errors.
- Introduction of moderate-density repeated burst error and study of cyclic codes detecting repeated burst errors and moderate-density repeated burst errors.
- Generalization of 2-repeated solid burst errors and study of linear codes those are capable of correcting such errors.

3. THESIS OUTLINE

The thesis contains six chapters and is organized as follows:

CHAPTER 1

Introduction & Relevant Literature Survey

This chapter is introductory in which a brief history of Coding Theory and background material relevant for investigations undertaken in later chapters as also some applications of coding theory are given. Different types of error patterns, in terms of which later studies are made, are described. Some well known bounds on number of parity checks and minimum distance of a code are stated.

Apart from general study of linear codes, a brief overview of cyclic codes, needed for our later study, is also given in the chapter. The chapter also includes, a brief overview of the work reported in the later five chapters.

CHAPTER 2

Results on Weights of Repeated Bursts of Length b

Codes being the mathematical tools for reliable communication over noisy channels, with the help of imbued redundancy, one major problem are to keep the redundancy minimum and improve efficiency. In the early studies, the case being binary, weight consideration in defining errors did not arise. These were studied with interest over the number of code vectors

of a code. Weight distributions, differently of course, have been deeply studied by McWilliams and Sloan [23] in the case of random errors. They studied codes with different weights. But efficiency requires close and critical study of constraints on the errors. In the case of burst errors beside length, the weight provides an important consideration. This is an important area of study for burst errors also, that we have undertaken for multiple burst errors. Our study in this chapter is concerned to codes with weight constraints.

With the wide and varied applications, coding is no longer for binary case as the channels are also of different types. Therefore nature of errors, in particular burst errors, which usually depends upon the type of channel used during the process of transmission. It has been observed that in very busy communication channels, bursts repeat themselves.

A *2-repeated burst* of length b is a vector of length n whose only non-zero components are confined to two distinct sets of b consecutive components the first and the last component of each set being non zero.

An *m -repeated burst* of length b is a vector of length n whose only nonzero components are confined to m distinct sets of b consecutive components, the first and the last component of each set being nonzero.

In Chapter 2, we study repeated bursts of a given length or less with and without weight constraints.

First we have obtained results on counting the numbers of 2-repeated bursts with and without weight consideration. Then we derived an expression for the total weight of all 2-repeated bursts in a vector of length n is found in this chapter. In our next result, we have given a recurrence relation between total weights of 2-repeated bursts of different lengths. In coding theory, minimum weight being of some importance, we have obtained an expression for the minimum weight of a vector having 2-repeated bursts in the space of all n -tuples. We have obtained an expression for the total weight of 2-repeated bursts of length b and weight w or less in the space of all n -tuples. A recurrence relation in this very general case is obtained. An upper bound on the minimum weight of a 2-repeated burst of length b with weight w or less is established.

As a generalization of 2-repeated bursts, we have considered m -repeated bursts and analogous results are obtained for m -repeated bursts in both cases with weight constraints and without weight constraints.

My two published research papers related to this chapter are:

- (i) BhuDev Sharma and **Barkha Rohtagi**, “Some results on weights of vectors having 2-repeated bursts,” *Cybernetics and Information Technologies*, vol. 11, no. 1, 36-44, 2011.
- (ii) BhuDev Sharma and **Barkha Rohtagi**, “Some results on weights of vectors having m -repeated bursts,” *Cybernetics and Information Technologies*, vol. 11, no. 3, 3-11, 2011.

CHAPTER 3

On Weights of Repeated Bursts of Length b (fixed) and Repeated Low-Density Burst of Length b (fixed)

When repeated bursts are introduced by impulse noise, as it happens in the case of large number of practical channels, the length of a repeated burst depends upon the duration of the impulse noise. Also, the number of errors, i.e., the density of errors within a repeated burst clearly depends upon the impulse noise. Depending upon the behavior of the channel, either due to impulse noise or otherwise, the density of errors in the repeated bursts introduced by the channel is either too small or too large. Employing the usual burst-correcting codes is not quite appropriate for this purpose of efficient transmission over such a channel. In such cases we need to consider low-density repeated bursts instead of simple repeated bursts.

In Chapter 3, our study is concerned with repeated burst error of length b (fixed). We start with obtaining results for simple repeated bursts of length b (fixed) and then we find results for repeated low-density bursts of length b (fixed). First we have obtained results for 2-repeated burst errors of length b (fixed). We have obtained results on counting the number of 2-repeated burst errors of length b (fixed) with and without weight constraints. An expression for total weight of 2-repeated bursts of length b (fixed) is derived. An upper bound on the minimum weight element of 2-repeated bursts of length b (fixed) is also obtained. Results regarding total weight and average weight of 2-repeated bursts of length b (fixed) with weight w are obtained. A recurrence relation between weights of different 2-repeated bursts is also obtained. There is a result on average weight of 2-repeated bursts of length b (fixed).

As a generalization of 2-repeated bursts of length b (fixed), we have considered m -repeated bursts of length b (fixed) and corresponding results are obtained for m -repeated bursts in the both cases with weight constraints and without weight constraints.

Further in an analogous manner, we have obtained results for total numbers of 2-repeated low-density bursts of length b (fixed), total weight of 2-repeated low-density bursts of length b (fixed) and minimum weight of 2-repeated low-density bursts of length b (fixed). After that we have obtained corresponding results for m -repeated low-density bursts of length b (fixed).

My research papers related to this chapter are:

- (i) BhuDev Sharma and **Barkha Rohtagi**, “On weights of 2-repeated bursts,” *Ratio Mathematica- Journal of Applied Mathematics, Italy, 2013.* (Accepted)
- (ii) BhuDev Sharma, **Barkha Rohtagi** and A.K Aggarwal, “Some results on weights of m -repeated bursts,” .(Communicated)

CHAPTER 4

Moderate-Density Repeated Burst Error Detecting Cyclic Codes

In this chapter we study cyclic codes detecting repeated burst errors and have considered a different kind of repeated burst, i.e., moderate density repeated burst. Then cyclic codes detecting moderate-density 2-repeated burst errors have been studied. Further, we obtained results for cyclic codes detecting m -repeated burst errors and moderate-density m -repeated burst errors. We then make a comparative study of results for moderate-density 2-repeated burst error of length b and moderate-density 2-repeated burst errors of length b (fixed). This study shows that there is a difference with regular interval between the values obtained for moderate-density 2-repeated burst error of length b and moderate-density 2-repeated burst errors of length b (fixed), when we apply similar weight constraints on them.

In this study, it is interesting to see that while obtaining results for moderate-density m -repeated burst errors, there arise two special cases:

- (i) For $m = 1$ and neglecting the consideration of starting positions, the result reduces to the case of moderate-density open-loop burst error in [20].
- (ii) For $b = 1$, the result reduces to the case of random errors.

This study may help us in developing codes which detect/correct those errors that are repeated bursts of a given length with weight lying between some pre assigned values. The development of such codes economizes in the parity-check digits required, suitably reducing the redundancy of the code, i.e., suitably increasing the efficiency of transmission.

My research papers related to this chapter:

- (i) BhuDev Sharma and **Barkha Rohtagi**, “Moderate-density 2-repeated burst error detecting cyclic codes”, *International Journal of Emerging Trends in Engineering and Development*, ISSN NO: 2249-6149, vol. 4, no. 2, pp.49-55, 2012.
- (ii) BhuDev Sharma and **Barkha Rohtagi**, “Moderate-density m -repeated burst error detecting cyclic codes”, *International Journal of Emerging Trends in Engineering and Development*, ISSN NO: 2249-6149, vol. 4, no. 2, pp.309-316, 2012.

CHAPTER 5

Linear Codes Correcting Repeated Solid Burst Errors

In situations, where channel introduces repeated burst errors in such a way that each digit in the error vector is nonzero, it is uneconomical to use simple repeated burst error correcting /detecting codes. Such types of repeated burst errors are termed as *repeated solid burst errors*.

In this chapter, we have generalized 2-repeated solid burst errors. Linear codes correcting repeated solid burst errors are studied. An example is given that shows that the syndromes of all repeated solid bursts of length 3 or less are nonzero and distinct.

My research papers related to this chapter:

- (i) **Barkha Rohtagi** and BhuDev Sharma, “Bounds on codes detecting and correcting 2-repeated solid burst errors,” (Accepted with minor revision in *Journal of Applied Mathematics and Informatics*).

CHAPTER 6

Conclusion and Future Scope

The work reported in Chapters 2 to 5, contains a variety of results on different kinds of multiple-bursts correcting codes. With use of coding through digital operations having widened, and the nature of errors being multiple bursts of one or the other kind, our study should be found helpful in closer examination of efficiency criteria.

From theoretical point of view, our study of multiple bursts in message vectors presents a mathematical formulation of a much broader and general class of errors unifying random errors and burst errors. We like to emphasize that our study of multiple burst errors unifies the study of random errors and burst errors. In random errors error positions are counted singly, the same when the error positions are not just single digits but bursts of certain length and type, our study develops.

Several classical and recently reported other studies in various journals easily arise as special and or particular cases of our results [5, 1, 14, 8, 9, 30, 20, 21 and7].

We seem to have conclusively developed a perspective when random error codes are a sub-class of our multiple bursts codes.

Also by considering bursts of all four different types, the study is very broad in its coverage. Our study provides repeated burst correction/detection criteria for different variety of channels that in practice can be used with currently ever widening phenomena of digital communication.

Results on cyclic considerations have lead to important class of codes. Continuing in that direction, we have introduced moderate-density repeated burst errors and obtained results regarding cyclic codes that are capable of detecting repeated burst errors and moderate-density repeated burst errors. Further, a comparative study between moderate-density open-loop burst and moderate-density repeated burst is also made. Also generalized is the notion of 2-repeated solid burst error.

In terms of specific concepts introduced and results obtained, we have introduced following new concepts and have obtained results depending on:

- i. We introduced and have used the concept of weight in multiple-bursts errors calculation of some pre defined repeated bursts, with and without weight constraints; this allowed the development of codes that may be proved more efficient than those are developed so far.
- ii. We have defined two different types of moderate-density repeated bursts and studied cyclic codes for detecting these errors. This study may be proved as a useful mathematical tool.
- iii. The generalization of 2-repeated solid burst errors, in itself is an area of quite importance for channels that are practically more in use such as supercomputers and

some space channels. The results obtained on number of parity check digits for codes correcting these error patterns are very useful in determining the efficiency of the code.

The work submitted provides leads to further research problems and areas. While the number could be quite large, we like to mention a few in the following:

- Study of cyclic codes detecting/correcting repeated burst errors with specific densities, i.e., in terms of low-density, high-density.
- Code construction for repeated burst errors in block wise manner.
- Development of Reed-Solomon type codes for multiple error correction and different type of constructions.
- Study of repeated bursts with weights other than Hamming weight, i.e., Lee weight, Sharma-Kaushik weight, Euclidean weight.
- Study of repeated burst errors of variable lengths.

**THE THESIS CONTAINS MATERIAL FROM
MY FOLLOWING RESEARCH PAPERS
(International Refereed Journals)**

1. BhuDev Sharma and **Barkha Rohtagi**, “Some results on weights of vectors having 2-repeated bursts,” *Cybernetics and Information Technologies*, vol. 11, no. 1, 36-44, 2011.
Indexed in: AMS Digital Mathematics Registry, SCOPUS, INSPEC.
2. BhuDev Sharma and **Barkha Rohtagi**, “Some results on weights of vectors having m -repeated bursts,” *Cybernetics and Information Technologies*, vol. 11, no. 3, 3-11, 2011.
Indexed in: AMS Digital Mathematics Registry, SCOPUS, INSPEC.
3. BhuDev Sharma and **Barkha Rohtagi**, “Moderate-density 2-repeated burst error detecting cyclic codes,” *International Journal of Emerging Trends in Engineering and Development*, ISSN NO: 2249-6149, vol. 4, no. 2, pp.49-55, 2012.
Indexed in: Google Scholar, Index Copernicus, NASA Advanced Lab (U.S.A.), EBSCO (U.S.A.), Science Central (U.S.A.).
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6. BhuDev Sharma, **Barkha Rohtagi** and A.K. Aggarwal., “Some results on weights of m -repeated bursts,” *Cybernetics and Information Technologies, 2013.* (Communicated).

7. **Barkha Rohtagi** and BhuDev Sharma, “Bounds on codes detecting and correcting 2-repeated solid burst errors,” (Accepted with minor revision in *Journal of Applied Mathematics and Informatics*)

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