INFORMATION MEASURES AND AGGREGATION OPERATORS ON FUZZY / INTUITIONISTIC FUZZY SETS WITH APPLICATIONS IN DECISION MAKING

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1. INTRODUCTION

In nature as well as otherwise there are what we can call deterministic and non-deterministic phenomena. Non-deterministic ones attract more challenges in scientific study. When the patterns of occurrence of non-deterministic phenomena are stable in their occurrence, their study was taken up in statistics. However, in real world, there is a large variety of vague phenomena, imprecise concepts and not well defined quantities, which fall beyond the scope of conventional precise mathematics. For such vague phenomena, the term 'Fuzzy' has been coined in mathematics. Systematically, the theory of ***fuzzy sets*** (FSs) was introduced by Zadeh [48] in 1965, as a generalization of classical set theory, for representing imprecise and vague phenomena. Fuzzy set theory allows objects to be members of the set with a degree of membership which can take any value in the closed interval $[0,\ 1]$. Zadeh [48] has proposed the following definition of a fuzzy set:

A *fuzzy set* $\tilde{A}$ defined in a discrete universe of discourse $X = \{x_1, x_2, ..., x_n\}$ is given as

$$\tilde{A} = \left\{ \langle x,\ \mu_{\tilde{A}}(x) \rangle \,\middle|\, x \in X \right\},$$

where $\mu_{\tilde{A}} : X \to [0,1]$ is the membership function of $\tilde{A}$. The number $\mu_{\tilde{A}}(x)$ describes the degree of membership of $x \in X$ in $\tilde{A}$.

Research on the theory of fuzzy sets has increased exponentially over the past 45 years; both within mathematics and in its applications. This ranges from traditional mathematical subjects such as algebra, analysis, calculus, logic etc. to information theory, graph theory, operation research, decision theory, coding theory, pattern recognition, automata theory, aggregation operators, cluster analysis etc. Consequently, fuzzy set theory and its generalizations have emerged as a potential area of interdisciplinary research.

As a generalization of Zadeh's fuzzy sets, Atanassov [1] introduced ***intuitionistic fuzzy sets*** (IFSs). An IFS is characterized by two functions expressing the degree of membership and the degree of non-membership, respectively. It is noted that IFSs can describe the fuzzy characters of the things more comprehensively and thus are a powerful and effective tool in dealing with fuzzy information.

Atanassov [1] has defined the intuitionistic fuzzy set as follows:

An *intuitionistic fuzzy set* A defined in a finite universe of discourse $X = \{x_1, x_2, ..., x_n\}$ is given as

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\},$$

where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$, with the condition $0 \le \mu_A(x) + \nu_A(x) \le 1, \quad \forall\ x \in X$.

The numbers $\mu_A(x)$ and $\nu_A(x)$ denote respectively the *degree of membership* and *degree of non-membership* of $x \in X$ in A. Over the last few decades, researchers have paid great attention to investigations in the IFS theory and its applications in many areas including decision making, medical diagnosis, and pattern recognition, market prediction, facility location selection etc.

On fuzzy and intuitionistic fuzzy set theory, our work is in the following two directions:

1. Information theoretic measures like as entropy, divergence and inaccuracy measures and their applications in decision making.
2. Aggregation operators and their applications in decision making.

INFORMATION THEORETIC MEASURES

We start with probabilistic approach and then take-up fuzzy and intuitionistic fuzzy approach.

Probabilistic Approach:

Entropy: The concept of *entropy* is extensively used in literature as a quantitative measure of uncertainty associated with a random variable. The development of the idea of entropy by Shannon [25], in his seminal paper '*A Mathematical Theory of Communication*', provided the beginning of a separate branch of learning namely the Information Theory. In his paper, Shannon set up mathematical schemes for quantitatively defining the concepts of information and proved a number of very general results with deeper consequences. The idea of entropy has since been widely used in different areas including communication theory, quantum information theory, finance, image registration, decision making etc. Generalizations of Shannon's entropy have been proposed by Rènyi [20], Havrda-Charvát [10], Sharma and Mittal [22], Sharma and Taneja [23] etc. In 1989, Pal and Pal [19] proposed another entropy function called '*exponential entropy*' and studied its applications in pattern recognition, image extraction, feature evaluation, and image enhancement.

Divergence: The idea of 'probabilistic divergence', which in some sense assesses how 'close' two probability distributions are from one another, has been widely applied in probability, statistics and information theory. *Divergence measure*, first introduced by Kullback and Leibler [16], is a measure of the extent to which the assumed probability distribution deviates from the true one. A generalization came about in 1961 in the work of Rényi [20], who introduced a generalized divergence measure of order-α. After that, a number of generalized divergence measures have been introduced by Havrda and Charvát [10], Sharma and Mittal [24], Taneja [27] among others. In 1991, Lin [17] pointed out some draw backs of Kullback and Leibler's divergence measure and proposed a modified divergence measure.

Inaccuracy: In 1961, Kerridge [15] firstly introduced the idea of *inaccuracy measure* in information theory as a generalization of Shannon's entropy. It is normal for an observed distribution to differ from a theoretical distribution of a random variable. Kerridge addressed to the problem of defining an (information theoretic) measure of inaccuracy of the distribution to the standard distribution under reference. Recently, Taneja et al. [28] proposed a dynamic version of Kerridge's inaccuracy to measure the inaccuracy between two residual life-time distributions. In addition, the concept of inaccuracy has its applications in statistical inference, estimation, probability distributions characterization and coding theory.

Fuzzy Approach:

Fuzzy Entropy: In Fuzzy set theory, the entropy is defined as a measure of fuzziness. The first attempt to quantify the fuzziness was made in 1968 by Zadeh [49], who based on probabilistic framework, introduced the fuzzy entropy by combining probability and membership function of a fuzzy event as weighted Shannon entropy [25]. In 1972, De Luca and Termini [8] formulated four axioms which the fuzzy entropy measure should comply with, and they defined the entropy of a fuzzy set based on Shannon's function. Later, many other researchers made more efforts in this particular area. For instance, Kaufmann [14] proposed fuzzy entropy of a fuzzy set by a metric distance between its membership function and the membership function of its nearest crisp set. Yager [42] defined an entropy measure of a fuzzy set in terms of the lack of distinction between fuzzy set and its complement. In 1989, Pal and Pal [19] proposed an entropy based on exponential function to measure the fuzziness called '*exponential fuzzy entropy*'. A number of parametric generalizations of De Luca and Termini's entropy measure have been studied by various researchers in last two decades. In 2007, Ding et al. [9] extended the idea of De Luca and Termini's fuzzy entropy for pairs of fuzzy sets and defined some new fuzzy information measures such as conditional fuzzy entropy, joint fuzzy entropy and fuzzy mutual information.

Fuzzy Divergence: In 1993, the idea of divergence measure was extended to fuzzy set theory by Bhandari and Pal [3]. Fuzzy divergence measure, proposed by Bhandari and Pal [3], gives a fuzzy information measure for discrimination of a fuzzy set $\tilde{A}$ relative to some other fuzzy set $\tilde{B}$. Afterwards, by using the idea of Lin [17], Shang and Jiang [21] introduced a modified version of fuzzy divergence measure proposed in [3] and applied it to multi-criteria decision making with fuzzy information. Hooda [11] proposed a generalized fuzzy divergence measure corresponding to Havrda and Charvat [10] divergence measure. These fuzzy divergence measures have found extensive applications in many areas including decision making, pattern recognition, medical diagnosis, fuzzy clustering etc.

Intuitionistic Fuzzy Approach:

Intuitionistic Fuzzy Entropy: Firstly, Burillo and Bustince [4] introduced the concept of entropy in intuitionistic fuzzy set theory, which allows us to measure the degree of intuitionism associated with an IFS. Szmidt and Kacprzyk [26] extended the axiomatic definition of fuzzy entropy given by De Luca and Termini to IFSs. Furthermore, based on the geometrical interpretation of IFSs, they also proposed a new entropy measure for IFSs. Vlachos and Sergiadis [29] proposed another measure of intuitionistic fuzzy entropy and revealed an intuitive and mathematical connection between the notions of entropy for fuzzy set and intuitionistic fuzzy set. Zhang and Jiang [50] defined a measure of intuitionistic fuzzy entropy for intuitionistic fuzzy sets by generalizing of the De Luca Termini [8] logarithmic fuzzy entropy. In last couple of years, many researchers also proposed different entropy functions for IFSs and applied them in different real life problems.

Intuitionistic Fuzzy Divergence: In 2007, Vlachos and Sergiadis [29] proposed the intuitionistic fuzzy divergence measure for the first time and studied its application pattern recognition and medical diagnosis. Further, Wei and Ye [32] proposed an improved version of intuitionistic fuzzy divergence in [29] and developed a method for pattern recognition with intuitionistic fuzzy information. Hung and Yang [12] defined another divergence measure called '*J*-divergence' for measuring the difference between two IFSs and then applied it to clustering analysis and pattern recognition.

AGGREGATION OPERATORS

Aggregation has come to be recognized as a very general rather new process of combining / fusing several numerical values in one representative value, and an aggregation operator performs this operation. These operators occur wherever aggregating data / values is necessary: pure mathematics (for example, functional equations, integration theory, theory of means and averages), applied mathematics (for example, probability theory, operations research, decision theory), computer and engineering sciences (for example, artificial intelligence, information theory, pattern recognition), economics and finance (for example, game theory, voting theory, decision making) as well as many other applied fields of physics and natural sciences. In general, aggregation operators are used for summarizing the information gathered from various sources, making decisions and so on. In the literature, many aggregation operators have been developed to aggregate numerical data such the weighted arithmetic average (WAA) operator [2], the weighted geometric mean (WGM) operator [2], the weighted harmonic mean (WHM) operator [2], the ordered weighted averaging (OWA) operator [44], the ordered weighted geometric (OWG) operator [38], and the generalized ordered weighted averaging (GOWA) operator [43] and so on.

However, in some situations, the input values take the form of fuzzy numbers and intuitionistic fuzzy numbers rather than real numbers due to the increasing complexity of the socio-economic environment, time pressure, not sufficient level of knowledge of the problem domain, and the ambiguity of human thinking. Therefore, how to aggregate fuzzy numbers and intuitionistic fuzzy numbers is an interesting and important research topic, which has received quite some attention from researchers and practitioners in last couple of years. To aggregate the fuzzy numbers and intuitionistic fuzzy numbers, a number of aggregation operators have been developed such as the fuzzy ordered weighted averaging (FOWA) operator [30, 39], the fuzzy ordered weighted geometric (FOWG) operator [6, 40], the fuzzy weighted harmonic mean (FWHM) operator [41], the fuzzy ordered weighted harmonic mean (FOWHM) operator [41], the fuzzy hybrid harmonic mean (FHHM) operator [41], the fuzzy generalized OWA (FGOWA) operator [18], the fuzzy induced ordered weighted harmonic mean (FIOWHM) operator [35], the intuitionistic fuzzy weighted geometric (IFWG) operator [36], the intuitionistic fuzzy ordered weighted geometric (IFOWG) operator [36], the intuitionistic fuzzy hybrid geometric (IFHG) operator [36], the intuitionistic fuzzy weighted averaging (IFWA) operator [37], the intuitionistic fuzzy ordered weighted averaging (IFOWA) operator [37], the intuitionistic fuzzy hybrid averaging (IFHA) operator [37], the generalized intuitionistic fuzzy weighted averaging (GIFWA) operator [51], the generalized intuitionistic fuzzy ordered weighted averaging (GIFOWA) operator [51], the generalized intuitionistic fuzzy hybrid averaging (GIFHA) operator [51], the induced intuitionistic fuzzy ordered weighted geometric (IIFOWG) operator [33], the dynamic intuitionistic fuzzy weighted geometric (DIFWG) operator [34], the intuitionistic fuzzy Einstein weighted average (IFEWA) operator [31], the intuitionistic fuzzy Einstein ordered weighted average (IFEOWA) operator [31] etc.

In 2008, Yager [45, 46] proposed a new aggregation operator called ***prioritized weighted average (PWA) operator*** and studied its applications in multiple attribute group decision making. The prominent characteristic of the PWA operator is that it takes into account prioritization among attributes and decision makers. Recently, using the idea of Yager's PWA operator, some aggregation operators have also introduced in the literature by researchers with fuzzy / intuitionistic fuzzy information [47, 52].

MULTIPLE CRITERIA DECISION MAKING:

Decision making is a process of selecting the best option (or options) from a finite number of feasible options. It is a very common activity present in almost every human functioning and plays an important role in business, finance, management, economics, social and political science, engineering and computer science, biology and medicine etc.

Multiple criteria decision making (MCDM) refers to select or ranking options from available options with respect to multiple, usually conflicting criteria in the presence of single decision maker or multiple decision makers. In general, multicriteria decision making can be classified as (a) multiple attribute decision making (MADM) and (b) multiple objective decision making (MODM).

MODM problems involve designing the best option given a set of conflicting objectives. A typical example is mathematical programming problems with multiple objective functions. In contrast to MODM problems, MADM involves in the evaluation, selection and ranking of options from available alternatives with respect to various criteria. In last few decades, the multiple criteria decision making has received a great deal of interest from researchers and practitioners in many disciplines.

In many real life decision making situations, the available information is vague or imprecise. To adequately solve decision problems with vague or imprecise information, fuzzy set theory and intuitionistic fuzzy sets theory have become powerful tools. In last two decades, several multiple criteria decision making theories and methods under fuzzy / intuitionistic fuzzy environment have been proposed for effectively solving the multiple criteria decision making problems and numerous applications have been reported in the literature [7, 12, 13, 18, 29-41, 44-47, and 50-52].

2. OBJECTIVE OF THE STUDY

In this age of technology and scientific theories for governance and management, mathematical tools and procedures are being increasingly used. Shannon's theory of communication that was developed for probabilistic phenomena, with generalizations served other areas, for example economics, commerce, decision theory, etc. This gave rise to study of uncertain or vague phenomena that are not probabilistic. Information type measures on fuzzy / intuitionistic fuzzy sets were introduced.

Objective of this study, in one part, is devoted to introduce new information theoretic measures on fuzzy / intuitionistic fuzzy sets. These measures have elegant properties that increase their reach and applicability. In the thesis, beside mathematical studies, algorithms for applications in decision making are developed and illustrated. .

Another allied area undertaken is that of aggregation operators, a rather new and very general way, including priority considerations of fusing available varied data in the form of fuzzy numbers / intuitionistic fuzzy numbers into one representative value.
The main objectives being mathematical study of fuzzy / intuitionistic fuzzy phenomena through information theoretic measures and aggregation operators, following specific points can be noted:

- Study some new information measures in fuzzy set theory and develop some decision making methods for solving multiple-criteria decision making problems with fuzzy information
- Consider some new information measures in intuitionistic fuzzy set theory and develop some decision making methods for solving multiple criteria decision making problems with intuitionistic fuzzy information
- Develop a prioritized weighted aggregation operator for fuzzy information based on Yager's PWA operator and study its applications in multiple attribute group decision making
- Introduce some new prioritized weighted aggregation operators for intuitionistic fuzzy information based on Einstein operations and study their applications in multiple attribute group decision making

3. THESIS OUTLINE

The thesis contains ten chapters and is organized as follows:

CHAPTER 1

INTRODUCTION

This chapter presents a brief literature survey on measures of entropy, divergence measures and measures of inaccuracy. It also outlines the basic concepts of fuzzy sets and intuitionistic fuzzy sets. A brief review on fuzzy aggregation operators and intuitionistic fuzzy aggregation operators are given here. In addition, a general overview of multiple criteria decision making problem is presented and it also summarizes the objectives with an overview of the work reported in later chapters.

CHAPTER 2

NEW EXPONENTIAL FUZZY ENTROPY MEASURES

In probabilistic setting, several generalizations of 'Shannon's entropy' have been considered to make the concept applicable in different situation. Exponential entropy, proposed by Pal and Pal [19], is an important non-formal generalization of Shannon's entropy which has found some advantages over Shannon's entropy in many applications.

In this chapter, we consider exponential entropy measure for pair of fuzzy sets. Firstly, we propose exponential fuzzy joint entropy, exponential fuzzy conditional entropy, and exponential fuzzy mutual information measure. Some properties and relations of proposed measures are proved. Further, we propose a parametric generalization of exponential fuzzy entropy called 'exponential fuzzy entropy of order-α' with its axiomatic justification. It is also shown that the De Luca and Termini's logarithmic fuzzy entropy and Pal and Pal' exponential fuzzy entropy are the special cases of our proposed generalized exponential entropy.

Matter of this chapter is based on my following research papers:

- **R. Verma** and B. D. Sharma, "On generalized exponential fuzzy entropy," *World Academy of Science, Engineering and Technology*, vol.5, pp.956-959, 2011, USA.
- **R. Verma** and B.D. Sharma, "Exponential information measures on pairs of fuzzy sets," *Revision Submitted to International Journal of Intelligent Systems and Applications*, 2014. **[Impact Factor: 0.100]**

CHAPTER 3

GENERALIZED FUZZY DIVERGENCE MEASURE AND ITS APPLICATION IN MULTIPLE CRITERIA DECISION MAKING

Divergence is an important measure in information theory as well as in fuzzy set theory which has widely used by researchers in many application areas. Generalized divergence measures provide flexibility to the users and enhance their applicability range.

This chapter proposes a new generalized fuzzy divergence measure, an extension of fuzzy divergence measure proposed by Shang and Jiang [21]. It may be remarked that the strength of a measure lies in its properties. The new measure has elegant properties proved in this chapter to enhance the employability of this measure. Special cases are also discussed. Further, a decision making algorithm is developed based on proposed generalized fuzzy divergence measure for solving multiple criteria decision making problems with fuzzy information. A numerical example is also considered to illustrate the flexibility of the proposed decision making approach.

Matter of this chapter is based on my following research paper:

- **R. Verma** and B. D. Sharma, "On generalized fuzzy divergence measure and its application to multi-criteria decision-making," *Revision Submitted to Soft Computing,* 2014. **[Impact Factor: 1.124]**

CHAPTER 4

FUZZY INACCURACY MEASURE AND ITS APPLICATION IN MULTIPLE CRITERIA DECISION MAKING

In this Chapter, we introduce a measure of inaccuracy of a fuzzy set $\tilde{B}$ with respect to another fuzzy set $\tilde{A}$. This fuzzy inaccuracy measure is an extension of Kerridge's inaccuracy measure to fuzzy set theory. A number of elegant properties of the proposed fuzzy inaccuracy are stated and proved. Relations with De Luca and Termini's fuzzy entropy, Shang and Jiang's fuzzy divergence measure and proposed fuzzy inaccuracy measure are derived. A weighted version of fuzzy inaccuracy is also given here. Further, a decision making algorithm is developed based on proposed weighted fuzzy inaccuracy measure for solving multiple criteria decision making problems with fuzzy information. Moreover, two numerical examples are considered to illustrate the proposed decision making approach.

Matter of this chapter is based on my following research papers:

- **R. Verma** and B. D. Sharma, "A measure of inaccuracy between two fuzzy sets", *Cybernetics and Information Technologies*, vol. 11, no. 2, pp. 13-23, 2011.
- **R. Verma** and B. D. Sharma, "A new inaccuracy measure for fuzzy sets and its application in multi-criteria decision-making," *International Journal of Intelligent Systems and Application*, vol. 6, no. 5, pp. 62-69, 2014. **[Impact Factor: 0.100]**

CHAPTER 5

EXPONENTIAL ENTROPY FOR INTUITIONISTIC FUZZY SETS

In Chapter 5, our study is concerned with measure of intuitionistic fuzzy entropy. We introduce exponential intuitionistic fuzzy entropy, an extension of exponential fuzzy entropy proposed by Pal and Pal [19]. Its axiomatic justification is also established here. Further, a numerical example is considered to compare the proposed intuitionistic fuzzy entropy measure with some existing well known intuitionistic fuzzy entropy measures.

Matter of this chapter is based on my following research paper:

- **R. Verma** and B. D. Sharma, "Exponential entropy on intuitionistic fuzzy sets," *Kybernetika*, vol. 49, no. 1, pp. 114-127, 2013. **[Impact Factor: 0.619]**

CHAPTER 6

NEW DIVERGENCE MEASURE FOR INTUITIONISTIC FUZZY SETS AND ITS APPLICATION IN MULTIPLE-CRITERIA DECISION MAKING

This chapter proposes a new measure of divergence for intuitionistic fuzzy sets called 'generalized intuitionistic fuzzy divergence'. This divergence measure is a generalization of intuitionistic fuzzy divergence measure proposed by Wei and Ye [32].

Several properties of the proposed divergence measure are stated and proved. Moreover, special cases are also discussed.

Further, a decision making algorithm is developed based on proposed divergence measure for solving multiple criteria decision making problems with intuitionistic fuzzy information. In addition, a numerical example is also presented to illustrate the flexibility of the proposed decision making approach.

Matter of this chapter is based on my following research paper:

- **R. Verma** and B. D. Sharma, "On generalized intuitionistic fuzzy divergence (relative information) and their properties," *Journal of Uncertain Systems*, vol. 6, no. 4, pp.308-320, 2012.

CHAPTER 7

INACCURACY MEASURE FOR INTUITIONISTIC FUZZY SETS AND ITS APPLICATION IN MULTIPLE-CRITERIA DECISION MAKING

This chapter extends the idea of inaccuracy from fuzzy sets to intuitionistic fuzzy sets and introduces a measure of inaccuracy of an intuitionistic fuzzy set B A with respect to another intuitionistic fuzzy set A. Some properties of the proposed intuitionistic fuzzy inaccuracy are stated and proved. A weighted intuitionistic fuzzy inaccuracy measure is also given. Further, a decision making algorithm is developed based on proposed weighted fuzzy inaccuracy measure for solving multiple criteria decision making problems with intuitionistic fuzzy information. Two numerical examples are considered to illustrate the effectiveness of the proposed decision making approach.

Matter of this chapter is based on my following research paper:

- **R. Verma** and B. D. Sharma, "A new measure of inaccuracy with its application to multi-criteria decision making under intuitionistic fuzzy environment," *Journal of intelligent and Fuzzy Systems*, 2013. (Online Published) **[Impact Factor: 0.936]**

CHAPTER 8

FUZZY GENERALIZED PRIORITIZED WEIGHTED AVERAGE OPERATOR AND ITS APPLICATION IN MULTIPLE ATTRIBUTE GROUP DECISION MAKING

Aggregation operators play very important role in many fields such as decision making, cluster analysis, and information retrieval etc. In the literature, several aggregation operators have been developed by researchers to aggregate numerical data in different situations.

In this chapter, we study prioritized weighted average operator with triangular fuzzy information. First we have defined a new prioritized weighted average operator called '*fuzzy generalized prioritized weighted average (FGPWA) operator*' for aggregating the triangular fuzzy numbers. A number of properties of proposed aggregation operator are stated and proved. The main advantage of the FGPWA operator is that it does not only take into account prioritization among the aggregated arguments but also has a flexible parameter. It also includes a wide range of fuzzy prioritized aggregation operators, such as the fuzzy prioritized weighted average (FPWA) operator, the fuzzy prioritized weighted geometric (FPWG) operator and the fuzzy prioritized weighted harmonic (FPWH) operator. Further, a FGPWA operator based algorithm is developed for solving triangular fuzzy multiple attributes group decision making problems in which the attributes and decision makers are in different priority levels. Finally, a numerical example is presented to illustrate proposed approach to multiple attribute group decision making with fuzzy information. Different values of the parameter are considered in the range $(-\infty, 0) \cup (0, \infty)$. Results obtained show that the parameter can be a good tool for accommodating the decision maker's preferences, which vary from one setup to another. Therefore, the proposed aggregation operator with parameter provides the decision makers more choices.

Matter of this chapter is based on my following research paper:

- **R. Verma** and B.D. Sharma, "Fuzzy generalized prioritized weighted average operator and its application to multiple attribute decision making," *International*

Journal of Intelligent Systems, vol. 29, no. 1, pp. 26-49, 2014. **[Impact Factor: 1.416]**

CHAPTER 9

INTUITIONISTIC FUZZY EINSTEIN PRIORITIZED WEIGHTED AVERAGE OPERATORS AND THEIR APPLICATION IN MULTIPLE ATTRIBUTE GROUP DECISION MAKING

In this chapter, our study is devoted to prioritized weighted average operator with intuitionistic fuzzy information. First, we have defined two new prioritized weighted aggregation operators such as the intuitionistic fuzzy Einstein prioritized weighted average (IFEPWA) operator and the intuitionistic fuzzy Einstein prioritized weighted geometric (IFEPWG) operator for aggregating intuitionistic fuzzy information and discuss their particular cases. Some properties of IFEPWA and IFEPWG operators are also studied. Further, we have developed an algorithm for multiple attribute group decision making based on the proposed operators under intuitionistic fuzzy environment in which the attributes and decision makers are in different priority levels. Finally, a numerical example is presented to illustrate proposed approach to multiple attribute group decision making.

Matter of this chapter is based on my following research paper:

- **R. Verma** and B. D. Sharma, "Intuitionistic fuzzy Einstein prioritized weighted operators and their application to multiple attribute group decision making," *Applied Mathematics and Information Sciences*, 2014. (Accepted) **[Impact Factor: 1.232]**

CHAPTER 10

CONCLUSIONS AND FUTURE SCOPE

The chapter presents the conclusions of this thesis, and recommendations for future work. This is followed by bibliography.

RESEARCH PAPERS ON MATERIAL IN THE THESIS
(International Refereed Journals)

1. **R. Verma** and B. D. Sharma, "On generalized exponential fuzzy entropy", *World Academy of Science, Engineering and Technology*, vol.5, pp.956-959, 2011, USA.
   Indexed in: International Science Index, EBSCO, ERA
2. **R. Verma** and B. D. Sharma, "A measure of inaccuracy between two fuzzy sets", *Cybernetics and Information Technologies*, vol.11, no.2, pp.13-23, 2011, Bulgaria.
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3. **R. Verma** and B. D. Sharma, "A new inaccuracy measure for fuzzy sets and its application in multi-criteria decision-making," *International Journal of Intelligent Systems and Applications*, vol.6, no.5, pp. 62-69, 2014, Hong-Kong.
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   Indexed in: Scopus, Mathematical Reviews
6. **R. Verma** and B.D. Sharma, "A new measure of inaccuracy with its application to multi-criteria decision making under intuitionistic fuzzy environment," *Journal of Intelligent and Fuzzy Systems,* 2014, USA. (Online Published)
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7. **R. Verma** and B.D. Sharma, "Fuzzy generalized prioritized weighted average operator and its application to multiple attribute decision making," *International Journal of Intelligent Systems*, vol.29, no.1, pp.26-49, 2014, USA.

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8. **R. Verma** and B. D. Sharma, "Intuitionistic fuzzy Einstein prioritized weighted average operators and their application to multiple attribute group decision making," *Applied Mathematics and Information Sciences*, 2014, USA. (Accepted)
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9. **R. Verma** and B. D. Sharma, "On generalized fuzzy divergence measure and its application to multi-criteria decision-making," *Revision Submitted to Soft Computing,* 2014. **[Impact Factor: 1.124]**
10. **R. Verma** and B.D. Sharma, "Exponential information measures on pairs of fuzzy sets," *Revision Submitted to International Journal of Intelligent Systems and Applications*, 2014.
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