NON-LINEAR STRESS ANALYSIS OF THICK-WALLED CIRCULAR CYLINDERS

Synopsis of the Thesis submitted in fulfillment of the requirements for the Degree of

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By

RICHA SHARMA

विद्या तत्व ज्योतिसमः

Department of Mathematics

JAYPEE INSTITUTE OF INFORMATION TECHNOLOGY
(Declared Deemed to be University U/S 3 of UGC Act)
A-10, SECTOR-62, NOIDA, INDIA

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SYNOPSIS

1. INTRODUCTION:

Mechanics, the science of forces and motions, has been serving mankind since the beginning of our civilization through its various engineering applications. These real life applications have been achieved by some important principles and assumptions of mechanics based on experimental results. There are various branches of applied mechanics such as Solid mechanics and Fluid mechanics etc. The mechanical behavior of materials provide information of the deformable bodies such as bars, plates, cylinders, disks and shells etc. The theory of deformable solids can be named as strength of materials or mechanics of deformable solids. Research in the field of solid mechanics is not only for the basic understanding of this field but also for the advancements of engineering techniques in this area. The properties of the materials which are used in different industries have been examined thoroughly for the betterment of design of various structures from the stand-points of both safety and economy. This subject is of vital importance in the design of various modern engineering structures such as missiles, rockets, aerospace and surface transportation vehicles, oil refineries, solar and atomic power plants, etc. This demand of industry can only be completed if there is a proper understanding of stresses, strains and mechanical properties of materials and therefore, having the capability of combining them using the theoretical approach. A designer will concentrate on feasible design of structure which is technologically practicable and economically viable. A designer is also interested in thorough knowledge of stresses at which permanent deformation begins in a material or risk of fracture is to be expected. In addition to this, designer will have to consider the possible deformation in machine parts exposed to high level of stresses. Deformation is an observable and a measurable physical phenomenon and is of prime interest for design engineers.

Since 1950, the ‘theory of elasticity’ for anisotropic materials has been continuously developed and enriched with newer investigations and research on the problems of general nature. Thereafter, the general theory has been formed on a rigorous scientific basis and a number of laws have been established on this theory. A review on development of the theory of elasticity shows that during the period of last two hundred years the theory has gradually progressed and laid into the field of mechanics, and considered as first foundation for the design of engineering structures. In the last several years, anisotropic materials have been widely used in various fields because they exhibit excellent static and dynamic behavior. The study of anisotropic materials has drawn the attention of scientists because of its various applications in engineering problems. The state of the art structures and machineries are generally made of composite materials, whose elastic behavior may be analyzed macroscopically with the help of theory of elasticity. According to generalized Hooke's law, there are thirty six material constants. Due to symmetry of the planes these thirty six material constants reduce to five material constants in case of transversely isotropic materials whereas two in case of isotropic materials. Nowadays issues in plastic state of matter are main point of interest in many branches of science and engineering. The scientific study of plasticity in materials first started in 1864. The scientists and engineers are interested in plastic flow of materials because it is related to forming processes of materials such as rolling, forging, bending, stretching and deep drawing etc. The design engineer is also interested in deformation behaviors of materials in order to avoid excessive deflection or distortion in machine parts and engineering structures. Creep is the gradual deformation of a material with time under constant loading. Particularly at elevated temperatures some materials are susceptible to this phenomenon and even under the constant load strains can increase continually until fracture. This form of fracture is particularly occurs in turbine blades, nuclear reactors, furnaces and rocket motors etc.

Classical approach of mechanics uses linear strain measures which neglect the non-linear transition region through which the yield occurs and the fact that creep and relaxation strains are non-linear in nature. In classical theory, different constitutive equations are used for elastic and plastic states, which are based on certain hypothesis that simplifies the problem to some extent. In the classical theory of elasticity, the displacements are assumed to be very small so that higher order terms of displacement gradients are neglected and therefore measure of strain becomes linear. Linearization of problems has advantages of existence, uniqueness and stability, whereas the disadvantage of linearization is that, it may not be able to represent all the changes occurring in a medium. Therefore non-linear terms are very much important in the transition state. Tresca, in 1868, assumed that there exists a 'mid-zone' between the elastic and plastic regions opposing Saint-Venant's two-zone theory. This idea is embodied in the remarks of Todhunter and Pearson's theory of elasticity and strength of materials (1893). Although majority of research workers ignored Tresca's 'mid-zone' theory for the sake of analytical convenience, however, it has long been felt that such a 'mid-zone' actually exists. According to classical theory of elasticity, the perfect elastic state is one extreme and ideal plastic state is another extreme. The response of majority of materials to applied boundary traction and body forces is in between these two extremes and it is physically impossible to draw a sharp line between the elastic and plastic states. It has been realized for long that the plastic yielding of an elastic material has asymptotic behavior and consequently there arises a necessity of giving a unified treatment that can describe both the behavior patterns under different physical conditions. A few more attempts have been made in this direction, notable of which are by Green [1], Thomas [2], Chakrabarty [3] and Seth [4-5].

The effect of elasticity in the plastic region was not considered by Levy-Mises. This short-coming of Levy-Mises was removed by Prandtl (1924) and Reuss (1930) in their papers. Then, Thomas (1954, 55) extended this theory and gave a new theory which deals with the case of combined constitutive equations of elastic and plastic state by introducing the idea of 'quick transition zone' and 'non-uniformity' respectively. Tresca, Stokes, Love and others has given the basic idea of transition problems in elasticity, plasticity or in fluid mechanics. Friedrichs (1955), explains how asymptotic phenomena occur in physical problems in his address to 'The American Mathematical Society'. He employed perturbation technique to such types of problems for very small regions called 'quick transition regions' but his technique cannot be used for global distribution phenomena. Green (1956) has developed a general theory of work-hardening for incompressible materials which was a special case of Truesdell's theory of hypo-elasticity and had shown that a yield condition can be implied as an asymptotic approach for infinite values of the strain. But none of the above mentioned authors have recognized transition state as separate state like that of elastic and plastic state and hence they did not consider the existence of constitutive equations in the transition region. Then a question arises, whether it is reasonable to assume, that properties of the medium in the elastic state can suddenly change to the plastic state or one can divide these two states by a non-differentiable, singular or discontinuous surface, known as yield surface. Most of the authors up to the present time are in common agreement that the situation is not so. A medium cannot change directly from state 'A' into state 'B' without passing through an intermediate state 'T'. In a large number of problems the states 'A' and 'B' may be treated as linear fields, but 'T' is essentially a non-linear field, since both 'A' and 'B' dovetail into each other in 'T'. This, in turn, has increased the need of using ad-hoc, semi-empirical laws, such as yield conditions, creep-strain laws, etc. These ad-hoc and semi-empirical laws are based on long experimental results, but very few authors have ever tried to justify these ad-hoc assumptions with analytical bases. Finally Seth [5] has considered this intermediate region (transition region) separately and developed a new theory named as transition theory for elastic-plastic and creep deformations. As explained by many authors that transition from one state to another is an asymptotic phenomena, Seth has argued that at transition the differential system governing the physical phenomena should attains some sort of criticality. Once these critical points or the transition points are identified, the solution at these points gives the solution at transition state. The transition points obtained from differential system may or may not correspond to any transition state. If r' and r be the distances of the point from axis of symmetry in an axi-symmetric cylinder, then elastic property of the material of cylinder breaks down when differential coefficient $\partial r' / \partial r$ is zero or infinity. Then material is said to be in transition region which is the mid zone between elastic and plastic region. The material is said to be in fully plastic state if it is incompressible (i.e., when Poisson's ratio approaches ½).

The ordinary measures used in classical mechanics are found insufficient to deal with transition state, so generalization of these measures is required to explain real physical phenomena in continuum mechanics. Seth has given generalized strain measures, which when combined with the transition point analysis of differential equation, not only eliminates the use of yield conditions and creep strain laws but also employs the same constitutive equations to give the solutions at elastic-plastic and creep states through some transition function. Seth has defined the generalized strain measure as $e = \left[\frac{1}{n}\left\{1-\left(\frac{l_0}{l}\right)^n\right\}\right]^m$, where '*n*' is the strain measure, '*m*' is the irreversibility index, l_0, l are the undeformed and deformed lengths of the rod respectively. Green, Cauchy, Hencky, Swainger and Almansi strain measures have been calculated for particular values of *n* from the generalized principal strain measure. The most important contribution of generalized measures is that they eliminate the need of using semi-empirical laws and jump conditions. It may also be noticed that, the plastic and creep stresses are both implied in these measures [8] and there is no need of assuming these states separately as in classical theory. Since the creep strain rate depends on the stresses and temperature at a given structural state, it is not the total strain, but the total rate of creep strain, which is significant. Therefore, it can be expected that a generalized measure concept, in which the two parameters are experimentally determined, may provide better understanding of the creep behavior. The nature of strain rate is different for each case in the four stages of creep. It increases in the second stage, becomes stationary in the third and further increases in the fourth stage. Since the deformation is non-linear therefore we should construct a global strain-rate measure which can be used in all the four stages of creep. Thus, a generalized strain measure is developed for this purpose and utilized in a large number of problems of continuum mechanics [5-8, 17-26].

When a material yields at any point, it is reasonable to expect that the material will also yield at its neighboring points, rather than remain in the elastic state which is completely against the plastic state of the nearby particles. As the plastic yielding of a material is the result of breaking down of its internal or macroscopic structure, this plastic yielding will be complete or partial depending on the existing physical conditions of the material. This further leads to the identification of two material states i.e. a transition state and plastic state.

There are various ways to explain how transition may occur from one state into another state:

(i) At transition, the differential system defining the elastic state should attain some criticality or singularity.

(ii) If we consider the plastic state as an image of the elastic state under the transformation $x^k = x^k\left(X^k\right)$, where (X, Y, Z), (x, y, z) being the co-ordinates of a point in the undeformed and deformed states, respectively. Then at transition, due to the relationship between elastic and plastic state the Jacobian of the transformation should be zero or infinite. This means that when transition occurs, one-to-one correspondence does not exist between elastic and plastic state.

When a deformable solid material is subjected to some external loading systems such as pressure or temperature, it has been observed that firstly solid deforms elastically. If the loading is continued, then material deforms plastically, and if loading is further continued, it gives rise to time dependent continuous deformation known as creep deformation. It may be possible that number of transition states may occur at the same critical point, so the transition function will have different values at this point. This critical point is known as a multiple point and each branch of it will correspond to a different state. In general, the material from elastic state can change into (i) plastic state, or (ii) creep state, or (iii) first to plastic state and then to creep and vice-versa, depending upon the nature of loading. All these final states are reached through a transition state.

Elastic-plastic and Creep stresses for thick-walled homogeneous and non-homogeneous cylinders under internal pressure have been analyzed by many authors [9-16]. All these authors considered yield criteria, jump conditions, linear strain measure to calculate the stresses by using concept of infinitesimal strain theory. Transition theory [7] does not require any of the above assumptions and thus solves more general problems using the concept of generalized strain measure [8]. This theory gives the well-known strain measures in addition to the stresses in plasticity and creep problems by determining the asymptotic solution at the transition points of the governing differential equations. It has been widely studied in the literature [17-26]. In the nuclear industry, cylinders made of functionally graded materials under internal and external pressure have become a point of interest due to their wide applications, for example, in steam generator tubes, in which primary coolant flows outside the tubes while secondary water flows inside the tubes. As permissible stress of any material is some proportion of the yield or ultimate stress of the material therefore it incorporates a 'safety factor' which provides a margin against the collapse condition in different types of thick-walled cylinders due to high pressure. Keeping in mind the various applications of thick-walled cylinders, we have performed the non-linear stress analysis of these cylinders under internal and external pressure with and without thermal effects.

2. OBJECTIVES OF THE STUDY:

The main objectives of the work reported in the thesis are:

- Mathematical modeling of the various non-linear problems of thick-walled cylinders.
- Elimination of need for assuming any classical yield conditions, creep-strain laws, etc.
- Identification of the 'transition state' as a separate state.
- Non-linear stress analysis of thick-walled circular cylinders under internal, external pressure and temperature.

3. CONTRIBUTION:

In this thesis, an attempt has been made to analyze some problems of technical importance on the basis of transition theory and generalized strain measure. Here, we have studied elastic-plastic and creep problems in isotropic, transversely isotropic, homogeneous and non-homogeneous materials using transition theory. It is observed that the asymptotic solution using the principal stresses leads from elastic to plastic state whereas the solution through principal stress difference method leads to creep state from elastic state [17]. For each transition point, we determine the stresses and results obtained have been analyzed numerically and depicted graphically also.

CHAPTER 1

The first chapter is introductory in which a brief history of the theory of elasticity, plasticity and creep is given. Transition theory has been discussed in details for finite deformation. A brief review of generalized strain measure has been presented. The generalized principal strain measure is defined as

$$e_{ii} = \int_0^{e_{ii}^A} [1-2e_{ii}^A]^{\frac{n}{2}-1} de_{ij}^A = \frac{1}{n}[1-(1-2e_{ii}^A)^{\frac{n}{2}}], \qquad (i, j = 1,2,3)$$

where n is the strain measure and e_{ii}^A is the principal Almansi finite strain components.

The generalized components of strain are defined as

$$e_{rr} = \frac{1}{n}[1-(r\beta'+\beta)^n], \quad e_{\theta\theta} = \frac{1}{n}[1-\beta^n], \quad e_{zz} = \frac{1}{n}[1-(1-d)^n], \qquad e_{r\theta} = e_{\theta z} = e_{zr} = 0,$$

where $\beta' = d\beta/dr$.

The stress-strain relation for thermo elastic isotropic material is

$$T_{ij} = \lambda\delta_{ij}I_1 + 2\mu e_{ij} - \xi\theta\delta_{ij}, \qquad (i\ , j = 1,2,3)$$

where $I_1 = e_{kk}$, T_{ij}, e_{ij} are strain invariants, stress tensors and strain tensors respectively, λ, μ are Lame's constants, δ_{ij} is Kronecker's delta, α being coefficient of thermal expansion, θ is temperature and $\xi = \alpha(3\lambda + 2\mu)$.

The stress-strain relations for transversely isotropic material are

$$T_{rr} = C_{11}e_{rr} + (C_{11} - 2C_{66})e_{\theta\theta} + C_{13}e_{zz}, \qquad T_{\theta\theta} = (C_{11} - 2C_{66})e_{rr} + C_{11}e_{\theta\theta} + C_{13}e_{zz},$$

$$T_{zz} = C_{13}e_{rr} + C_{13}e_{\theta\theta} + C_{33}e_{zz}, \qquad T_{zr} = T_{\theta z} = T_{r\theta} = 0,$$

where C_{ij}'s are material constants.

CHAPTER 2

Second chapter is divided into two sections in which first section deals with “Stress analysis of elastic-plastic functionally graded thick-walled cylinder under internal and external pressure”. In this section, our aim is to determine transitional and fully plastic stresses for thick-walled cylinder under internal and external pressure using Seth’s transition theory. The cylinder is made of functionally graded material with internal and external radii ‘a’ and ‘b’ respectively. The constitutive equations for both transition and fully plastic states are derived from the governing equations. It has been shown that the asymptotic solution through principal stresses leads from elastic to plastic state. It is observed that initial yielding in the functionally graded cylinder starts at $r=\left(e^{2}b^{-k}\right)^{-1/k}=r_1,\ k>0.$ From the numerical discussion, it is concluded that functionally graded cylinder with internal and external pressure is on the safer side of the design as compared to homogeneous cylinder because functionally graded cylinder requires high pressure for initial yielding as compared to homogeneous cylinder. It has also been concluded that highly compressible functionally graded cylinder is on the safer side of the design as compared to less compressible functionally graded circular cylinder because highly compressible cylinder requires high pressure for initial yielding as compared to less compressible functionally graded cylinder which leads to the idea of ‘Stress Saving’ that minimizes the possibility of fracture of cylinder.

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Second section deals with “Thermal stress analysis of elastic-plastic functionally graded thick-walled cylinder under internal and external pressure”. In this section, we consider thermal effects along with all other parameters which were considered in first section. Here, our aim is to determine the thermal stresses for functionally graded thick-walled circular cylinder under internal and external pressure using generalized Lebesgue strain measure. The stresses are calculated in both transition and fully plastic states. The constitutive equations for transition and fully plastic states are derived. It has also been shown that the asymptotic solution through principal stresses leads from elastic to plastic state. From the analysis of effective pressure, it is concluded that highly compressible functionally graded circular cylinder with temperature is on the safer side of the design as compared to less compressible functionally graded cylinder because highly compressible circular cylinder requires very high effective pressure to yield and then to become fully plastic as compared to less compressible cylinder. From the analysis of external pressure, it is concluded that highly compressible functionally graded circular cylinder with thermal effects is on the safer side of the design as compared to less compressible circular cylinder because highly compressible circular cylinder requires very high external pressure without internal pressure to yield and then to become fully plastic as compared to less compressible circular cylinder.

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CHAPTER 3

This chapter deals with "Finite thermal creep deformation in functionally graded thick-walled circular cylinder under internal and external pressure". In this chapter, our aim is to determine thermal creep stresses for functionally graded thick-walled circular cylinder under internal and external pressure using Seth's transition theory. Here, we have considered a thick-walled circular cylinder of internal and external radii '*a*' and '*b*' respectively, subjected to internal and external pressure with temperature applied at the internal surface. It has been shown that the asymptotic solution through principal stress difference leads from elastic to creep state. From the analysis of this problem, it has been concluded that circular cylinder with temperature is on the safer side of the design as compared to circular cylinder without temperature because circumferential stresses are less for circular cylinder with temperature as compared to circular cylinder without temperature. It has been noticed that circular cylinder with non-linear measure is on the safer side of the design as compared to cylinder with linear measure. Also, highly compressible circular cylinder is safe for designing as compared to less compressible circular cylinder because less compressible circular cylinder is having high circumferential stresses as compared to highly compressible circular cylinder. Thus we can conclude that by introducing a suitably chosen temperature gradient, the stresses due to pressure may be reduced, which leads to the idea of 'Stress Saving'. It means that introduction of thermal effects minimizes the possibility of fracture of cylinder.

(Multidiscipline Modeling in Materials and Structures, vol. 9(4), pp. 499-513, 2013. Indexed in SCOPUS, Impact Factor = 0.59, H-Index = 7).

CHAPTER 4

This chapter deals with "Stress analysis of transversely isotropic thick-walled cylinder under internal and external pressure". In this chapter, we have considered transversely isotropic thick-walled circular cylinder under internal and external pressure with internal radius 'a' and external radius 'b' respectively. Here, our aim is to determine the stresses in transversely isotropic thick-walled circular cylinder under internal and external pressure so that collapse of cylinder due to pressure can be avoided. The constitutive equations and stresses are obtained in transition and fully plastic states. It has also been shown that the asymptotic solution through principal stresses leads from elastic to plastic state. It is observed that initial yielding starts at internal surface. From the analysis, it can be concluded that circular cylinder under internal and external pressure made up of transversely isotropic material (Beryl) is on the safer side of the design as compared to the cylinder made up of isotropic material (Steel) as well as of transversely isotropic material (Magnesium). The main reason is that the percentage increase in effective pressure required for initial yielding to become fully plastic is high for Beryl as compared to Steel and Magnesium which leads to the idea of "Stress Saving" that reduces the possibility of collapse of thick-walled cylinder due to internal and external pressure.

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CHAPTER 5

This chapter is divided into two sections in which first section deals with the problem of "Stress analysis in torsion of a functionally graded thick-walled circular cylinder under external pressure". Here, we have considered a functionally graded thick-walled circular cylinder with internal radius 'a' and external radius 'b' respectively, subjected to an external pressure p. Radial, circumferential and shear stresses are calculated in transition and fully plastic states. The constitutive equations for transition and fully plastic states are also derived from the results. It has also been shown that the asymptotic solution through principal stresses leads from elastic to plastic state. From the numerical discussion, it is concluded that non-homogeneous cylinder with less compressibility is on the safer side of the design as compared to highly compressible non-homogeneous cylinder. This is because of the reason that shear stresses are less for less compressible non-homogeneous cylinder as compared to highly compressible non-homogeneous cylinder.

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The second section deals with "Torsion of a functionally graded thick-walled circular cylinder under internal and external pressure with steady state temperature". In this section, we have considered a functionally graded thick-walled circular cylinder with internal radius 'a' and external radius 'b' respectively, subjected to internal and external pressure with temperature at the internal surface. We determine stresses for transitional and fully plastic states. Results have been discussed numerically and depicted graphically. It has been concluded from numerical results that in case of torsion cylinder made of less compressible non-homogeneous cylinder without thermal effects is on the safer side of design as compared to the cylinder made of highly compressible non-homogeneous cylinder with or without thermal effects because shear stresses are minimum for less compressible non-homogeneous cylinder without thermal effects as compared to highly compressible non-homogeneous cylinder.

(Communicated to Multidiscipline Modeling in Materials and Structures, Indexed in SCOPUS, Impact Factor = 0.59, H-Index = 7).

CHAPTER 6

This chapter deals with "Elastic-plastic analysis of homogeneous thick-walled circular cylinder with internal and external pressure under steady state temperature". The objective is to identify the transition state and to analyze the behaviour of the stresses in the transition state. The transition state is identified by the critical points of the differential system governing the elastic state. Here, we have considered a thick-walled circular cylinder of internal and external radii 'a' and 'b' respectively, subjected to internal and external pressure with temperature θ_0 applied at the internal surface. It has been noticed that initial yielding starts at the internal surface. From the analysis, it is observed that less compressible circular cylinder with steady state temperature is on the safer side of the design as compared to highly compressible circular cylinder without thermal effects because pressure required for initial yielding is less for highly compressible circular cylinder as compared to less compressible homogeneous circular cylinder. Thus, introduction of temperature gradient reduces the possibility of fracture of cylinders under internal and external pressure.

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CHAPTER 7

This chapter is divided into two sections. First section deals with the problem of "Thermal creep transition of homogeneous thick-walled cylinder under external pressure". In this section, we determine the creep stresses and creep strain rates in thick-walled cylinder under external pressure with steady state temperature. Here, we consider a thick-walled circular cylinder of internal and external radii '*a*' and '*b*' respectively, subjected to external pressure with steady state temperature applied at the internal surface. It has been shown that the asymptotic solution through principal stress difference leads from elastic to creep state. Results have been discussed numerically and depicted graphically. From the analysis, it is concluded that less compressible thick-walled circular cylinder with thermal effects under external pressure with non- linear measure is on the safer side of design as compared to highly compressible thick-walled cylinder without thermal effects. Thus, introduction of temperature gradient reduces the possibility of fracture of cylinders under internal and external pressure.

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Second section deals with "Thermal creep transition of homogeneous thick-walled cylinder under internal and external pressure". In this section, our main aim is to determine thermal creep stresses and strain rates for homogeneous thick-walled circular cylinder under internal and external pressure using generalized Lebesgue strain measure. We also obtain the constitutive equations corresponding to the transition state and creep state. Here, we consider a thick-walled circular cylinder of internal and external radii 'a' and 'b' respectively, subjected to internal and external pressure with steady state temperature applied at the internal surface. Thermal creep stresses and strain rates have been determined and discussed numerically. It is concluded that less compressible thick-walled circular cylinder with thermal effects under internal and external pressure with non-linear measure is on the safer side of the design as compared to highly compressible thick-walled cylinder without thermal effects. This is because of the reason that less compressible thick-walled circular cylinder with thermal effects under internal and external pressure with non-linear measure is having less circumferential stresses as compared to highly compressible thick-walled cylinder without thermal effects.

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CHAPTER 8

Conclusion and Future scope of the study are presented in this chapter. Different types of thick-walled circular cylinders i.e. homogeneous, non-homogeneous, transversely isotropic cylinders and cylinders under torsion are considered for our studies in this thesis. The concept of transition theory is applied to find the transitional, plastic and creep stresses in different types of cylinders under internal pressure, external pressure and temperature. The transition points and asymptotic solution corresponding to these points are obtained. As we know that the permissible stress of any material is some proportion of yield or ultimate strength of the material which incorporates the safety factor. This safety factor provides the margin against collapse in cylinders under different loadings and reduces the possibility of fracture of cylinders due to internal and external pressure. From the analysis of effective pressure and circumferential stresses, it is concluded that in general non-homogeneous cylinders are more suitable for designing as compared to homogeneous cylinders. Also transversely isotropic cylinders are on safer side of design as compared to cylinders made up of isotropic materials. In future, the concept of transition theory can be extended for cylinders made up of orthotropic materials, nano-materials and micro-polar materials under different types of loadings.

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Author’s List of Publications:

1 Aggarwal A.K., **Sharma R.,** Sharma S., “Collapse Pressure Analysis of Transversely Isotropic Thick-walled Cylinder using Lebesgue Strain Measure and Transition Theory”, The Scientific World Journal, vol. 2014, dx.doi.org/10.1155/2014/240954, pp. 1-10, 2014.
Impact Factor: 1.73, Indexing: SCIE, SCOPUS, H Index: 40, H5 Index: 35. Peer reviewed: Yes, Publisher: Hindawi (New York, London and Cairo, Egypt).

2 Aggarwal A.K., **Sharma R.,** Sharma S., “Safety Analysis using Lebesgue Strain Measure of Thick-Walled Cylinder for Functionally Graded Material under Internal and External Pressure”, The Scientific World Journal, vol. 2013, dx.doi.org/10.1155/2013/676190, pp. 1-10, 2013.
Impact Factor: 1.73, Indexing: SCIE, SCOPUS, H Index: 40, H5 Index: 35, Peer reviewed: Yes, Publisher: Hindawi (New York, London and Cairo, Egypt).

3 Aggarwal A.K., **Sharma R.,** Sharma S., “Safety Analysis of Thermal Creep Non-Homogeneous Thick-Walled Circular Cylinder under Internal and External Pressure using Lebesgue Strain Measure”, Multidiscipline Modeling in Materials and Structures, vol. 9(4), pp. 499-513, 2013.
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4 **Sharma R.,** Aggarwal A.K., Sharma S., “Collapse Pressure Analysis in Torsion of a Functionally Graded Thick-Walled Circular Cylinder under External Pressure”, Accepted, Elsevier’s Procedia Engineering, 2014.
Impact Factor = 0.36, Indexing: SCOPUS, H Index: 11, H5 Index: 19, Peer reviewed: Yes, Publisher: Elsevier (Netherlands).

5 Aggarwal A.K., **Sharma R.,** Sharma S., “Safety Factors in Thermal Creep Transition of Thick-Walled Circular Cylinder under Internal and External Pressure using Transition Theory”, Accepted, International Journal of Applied Mathematics and Mechanics, 2014.
Indexing: Zentralblatt, Peer reviewed: Yes, Publisher: GBS (India).

6 Aggarwal A.K., **Sharma R.,** “Safety factors in Thermal Creep Transition of Thick-walled Circular Cylinder under External Pressure using Lebesgue Strain Measure and Transition Theory”, First International Conference on Structural Integrity (ICONS-2014), Kalpakkam, India, pp. 707–714, February 4-7, 2014.
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(Dr. A.K. Aggarwal)
Supervisor

(Richa Sharma)
Research Scholar