NON-LINEAR MHD FLOW PROBLEMS OF MICROPOLAR FLUIDS

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By

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SYNOPSIS

INTRODUCTION

Mechanics of non-Newtonian fluids has been one of the most challenging and interesting research areas for engineers, physicists, mathematicians and numerical simulists. In the past few decades, the interest in non-Newtonian fluids has increased tremendously, mainly due to its association with applied sciences and its indispensible role not only in theory but also in many industrial applications. Experimentally, it has been shown that the physical behaviour of these fluids significantly differs from that of the classical or Newtonian fluids, and thus falls outside the coverage of classical field theories. This inadequacy of the classical continuum approach to describe the complex mechanical behaviour of such fluids has therefore led to the developments of various microcontinuum theories such as simple microfluids, simple deformable directed fluids, micropolar fluids, polar fluids, dipolar fluids and anisotropic fluids. Notable among them is the micropolar fluid theory of Eringen [1]. Micropolar fluids are a subclass of micromorphic fluids (or “simple microfluids”) [2] in which the deformation of the fluid microelements is ignored but microrotational effects are present. The micropolar fluid theory constitutes an important branch of non-Newtonian fluid dynamics and is considered to be one of the best-established theories for fluids with microstructure. This theory provides a mathematical model for accurately simulating the flow characteristics of polymeric additives, food stuffs, detergents, gel propellants, colloidal suspensions, liquid crystals, lubricants, bubbly liquids, paints, physiological fluids (blood), smoke-laden air and geological flows containing suspended sediments.

Classical theory of continuum mechanics considers a fluent media as a dense aggregate of continuum particles which does not possess any internal structure. However, nature is abound with complex fluids which point to the need for incorporating micromotions into mechanics. Experiments due to Hoyt and Fabula [3] and Vogel and Patterson [4] with fluids containing minute amount of polymeric additives indicate a reduction in skin friction near a rigid body when compared with the skin friction in the same fluid without additives. Drag reduction in fluids containing polymers is a well documented phenomenon which cannot be explained on the basis of classical continuum mechanics. Eringen [2] introduced the theory of microfluids. By definition, a simple microfluid is a fluent medium whose properties and behaviour are affected by the local motion of the material particles contained in each of its volume element. These fluids can support stress moments, body moments and possess local spin inertia. However, the theory is complicated as a problem dealing with simple microfluids must be formulated in terms of a system of nineteen partial differential equations in nineteen unknowns and the underlying mathematical problem is therefore not so easily amenable to solution. Eringen simplified the microfluids to micropolar fluids [1]. The micropolar theory possesses adequate mathematical simplicity to make the engineering problems tractable. It takes into account the effects arising from particle micromotion. The particles called microelements are regarded as rigid and their rotation permits the presence of surface and body couples. Two independent vector fields, the translational velocity field and the microrotational field characterize the mechanical state of such fluids. The microrotation vector represents the rigid rotational motion of the particles contained in a small volume element about the centroid of the volume element. This local rotation of the particles is in addition to the usual rigid body motion of the entire volume element. None of these new effects are present in classical field theories of fluids.

Physically, viscous fluids consisting of rigid randomly oriented (or spherical) particles can be properly represented by the micropolar fluid model. Eringen has stated that the phenomena of drag reduction observed in fluids containing minute amount of polymeric additives can be explained by the theory of micropolar fluids. Papautsky *et al*. [5] found that a numerical model for water flow in microchannels based on the theory of micropolar fluids gave better predictions of experimental results than those obtained using the Navier–Stokes equation. Eringen [6] has also extended the microfluid theory to include the thermal effects, i.e. heat conduction, heat convection and heat dissipation, which is called the theory of thermomicrofluids. Kazakia and Ariman [7] extended the theory of micropolar fluids to introduce the heat conducting micropolar fluids.

The difficulties of solving the field equations even for Newtonian fluids are well known. The requirement of simplifying the full equations of motion therefore becomes necessary. The theory developed is commonly known as Prandtl's boundary layer theory and is primarily based on Prandtl's concept that the effect of viscosity is predominant only in a very thin layer close to the wall where (a) gradients of velocity field normal to the wall are much larger as compared to those parallel to the wall; (b) velocity component parallel to the wall in the flow direction is an order of magnitude higher as compared to other components of velocity. The development of boundary layer theory has been an extremely significant step in fluid dynamics and has found varieties of engineering applications. The

Navier-Stokes' equations are greatly simplified to a mathematically tractable form which are called as boundary layer equations. The knowledge of boundary layer theory helps in a better design of airfoils, wings, wind tunnel contours, supersonic nozzles, control surfaces etc. These processes of simplifications have later found extensive use in micropolar fluid flows. The boundary layer concept in micropolar fluids was first studied by Peddieson and McNitt [8] and later by Willson [9]. Further studies of boundary layer flow in micropolar fluids in different configurations include [10-14].

The theory of micropolar fluids has been a field of active research in fluid mechanics, for nearly five decades and has seen an incredible interest emerge in applications of the micropolar model to numerous problems in engineering sciences. The diverse areas to which the micropolar theory has been applied, include lubrications to human joints, blood flow in porous bio materials, cross diffusion dialysis flows, sediment transport in rivers, application in power generators, refrigeration coils, transmission lines, electric transformers, aeronautics and submarine navigation, cooling of electronic equipments, MHD accelerators, refrigeration coils, transmission lines, nuclear power plants, propulsion devices for aircraft, purification of crude oil, polymeric fluids, continuous casting of glass fiber, porous media, paper production and metal extraction. A comprehensive review of the subject and its applications can be found in the review articles by Ariman *et al*. [15, 16], in the books of Eringen [17], Lukaszewicz [18] and recently, Bég *et al*. [19].

In recent years, multi-physical micropolar flows have particularly emerged as a robust area of interest. Such flows involve magnetic, multi-mode heat and mass transfer, porous media, chemical reaction and other effects. Subsequently, there has been a significant advance in generalizing such fluid flow problems in different geometries with different physical conditions to various situations of practical interest.

Magnetohydrodynamics (MHD) is the study of the motion of electrically conducting fluids, namely, plasmas, liquid metals, salt water or electrolytes etc., in the presence of a magnetic field. The field of MHD was initiated by Hannes Alfvén. They are routinely used in industry to heat, pump, stir and levitate liquid metals. Many natural phenomena and technological problems are susceptible to MHD analysis. This subject has attracted numerous scientists and engineers for the last few decades because of its fascination and importance in various technological devices and in understanding the diverse cosmic phenomena. MHD phenomena are outcome of mutual interaction between magnetic field and electrically conducting fluid flowing across it i.e., electric current induced in the fluid as a result of its motion modify the field, and at the same time their flow in the magnetic field produces mechanical force called Lorentz force which modifies the motion. The set of equations which describes MHD is a combination of the Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism. These differential equations have to be solved simultaneously, either analytically or numerically. Maxwell’s equations describe the properties of electric and magnetic fields and connect them to their sources, charge density and current density. Many studies involving magnetic effect have been analyzed by many researchers [20-23].

Heat transfer is energy in transit which occurs due to temperature difference. The science of heat transfer is concerned with the analysis and estimation of the rate of heat transfer. The transfer of heat by combined forced and free convection is very common, but only in recent years attention is placed on determining the characteristics of such systems. Mixed convection flow of an electrically conducting fluid over a surface in the presence of a magnetic field is also of special technical significance because of its frequent occurrence in many industrial applications such as cooling of nuclear reactors, MHD marine propulsion, electronic packages, microelectronic devices etc. Many engineering processes occur at high temperatures and the knowledge of radiative heat transfer has become important for the design of pertinent equipments. The effect of thermal radiation on the forced and free convection flows is important in the context of space technology and processes involving high temperatures. Gas turbines, nuclear power plants, thermal energy storages and various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. Numerous studies investigating these aspects of fluid flows have therefore been reported [24-30].

Combined heat and mass transfer phenomena is found everywhere in nature and is important in all branches of science and technology. In several practical applications, there exist significant temperature and concentration differences between the surface of the hot body and the free stream. It is a major area of interest and has assumed practical importance in engineering devices such as heat exchangers, solar collectors, nuclear reactors and electronic equipments. Combined heat and mass transfer flows with chemical reactions have stimulated extensive research in science and technology in the past few decades due to numerous applications in drying processes, combustion processes, metallurgical flows, cooling towers and chemical engineering processes [31]. Significant analysis of electrically conducting convective flows with chemical reaction, have also received attention owing to diverse applications. In recent years, many authors [32-40] have explored various aspects of heat and mass transfer in such types of flows.

Convective transport phenomenon in porous media has been the subject of investigation in various scientific and technical fields. It is of fundamental importance in many technological processes such as in extraction of geothermal energy, cooling of nuclear reactors and underground disposal of nuclear wastes, petroleum reservoir operations, building insulation, irrigation systems, cooling of electronic components and the spreading of chemical pollutants in saturated soil and so on. Porous media abound in chemical engineering systems and magnetic fields are frequently used to control transport phenomena in electrically-conducting fluent media. In this regard, a significant amount of research work under different conditions and in the presence of various physical effects has been reported [41-46].

OBJECTIVES

The problems in micropolar fluids are challenging as one has to deal with coupled non-linear differential equations. Therefore the objectives of the work reported in the thesis are to

- develop a mathematical model for non-linear MHD boundary layer flow problems of micropolar fluids.
- obtain numerical solution for these equations using quasilinearization and finite element method.
- analyse the flow pattern by studying the parameters which affect the fluid flow.
- find the parameters that are responsible for the reduction of skin friction.
- identify the parameters which increase the rate of heat transfer and rate of mass transfer.

BRIEF CONTENTS OF THE THESIS

In the present thesis, micropolar fluid dynamics has been employed to study the steady, laminar, two-dimensional incompressible, mixed convection, MHD fluid flows in different geometries under various physical situations. Using similarity transformations, the governing differential equations of the problems considered, are transformed into system of nonlinear ordinary differential equations. A finite element solution [47] is presented to examine the effect of various key physical parameters which control the flow regime. The effects of these thermophysical parameters on velocity, microrotation, temperature and concentration functions are depicted graphically. Numerical results for the skin friction coefficient, couple stress, rate of heat transfer and rate of mass transfer are also computed. Computations have been compared with previously published results and are found to be in excellent agreement for all the problems considered. Additionally, mesh independence of the finite element computations is also achieved.

The present thesis is divided into seven chapters and the chapter wise summary is given as follows

Chapter 1: This chapter is an introductory chapter and presents a brief introduction to the dynamics of micropolar fluids. In this chapter, a review of the earlier works done by various researchers involving magnetic effect, multi-mode heat transfer, mass transfer, porous media, chemical reaction and other effect has been discussed. Additionally, a concise outline of the numerical method employed namely, the finite element method is also included.

Chapter 2: This chapter is divided into two sections and studies two different types of MHD micropolar fluid flow problems. The first section examines the influence of thermal radiation on mixed convection magneto-micropolar fluid flow from a vertical plate embedded in a non-Darcian porous medium with convective surface boundary condition. The effect of radiation parameter, magnetic parameter, buoyancy parameter, permeability parameter and convective parameter on velocity, microrotation (microelement angular velocity) and temperature functions are presented graphically and elucidated in detail. The study is relevant to high temperature materials processing in the chemical engineering industry.

In the second section, a mathematical model is developed to study the effects of thermal radiation and uniform transverse magnetic field on the mixed convective flow of a micropolar fluid near the stagnation point on a heated vertical permeable plate, in the presence of constant suction velocity. Effect of the important parameters namely, buoyancy parameter, magnetic parameter and radiation parameter on the velocity, micro-rotation and temperature functions have been shown graphically. The contribution of this study could be very useful for many engineering as well as industrial purposes where thermal radiation plays a significant role such as in MHD generators, plasma studies, solar collectors, geothermal energy extractions, metallurgical processes, nuclear reactors industry and polymer industries.

Chapter 3: It is also divided into two sections and discusses two different types of MHD micropolar fluid flow problems. In the first section, we examine the influence of thermal radiation on mixed convection dissipative magneto-micropolar fluid flow past a continuously moving plate with a convective surface boundary condition. The influence of radiation parameter, magnetic parameter and convective parameter on velocity, microrotation and temperature functions is elucidated in detail. Validation of the computations has been included with previously published results for special cases including non-magnetic and non-radiative studies. The study finds applications in electro-conductive polymer processing.

In the second section, a mathematical study is presented for the collective influence of buoyancy parameter, convective boundary parameter and temperature dependent viscosity on the steady mixed convective laminar boundary flow of a radiative magneto-micropolar fluid adjacent to a vertical porous stretching sheet embedded in a Darcian porous medium. The fluid viscosity is assumed to vary as an inverse linear function of temperature. The results obtained are depicted graphically to illustrate the effect of the various important controlling parameters on velocity, micro-rotation and temperature functions. The study finds applications in conducting polymer flows in filtration systems, trickle bed magnetohydrodynamics in chemical engineering, electro-conductive materials processing etc.

Chapter 4: In this chapter, a problem of the steady, two-dimensional, heat and mass transfer flow of a chemically reacting mixed convection magneto-micropolar fluid over a wedge with a convective surface boundary condition is investigated. Using similarity transformations found by Falkner and Skan [48], the governing transport equations are reduced to a system of nonlinear ordinary differential equations which are solved by employing the extensively-validated finite element method. The results obtained are depicted graphically to illustrate the influence of the various pertinent parameters on velocity, micro-rotation, temperature and concentration functions. Additionally, skin friction coefficient, local Nusselt number and local Sherwood number are also computed and presented graphically for the flow regime. The numerical solutions are compared with earlier studies for special cases of the wedge angle parameter and found to be in excellent agreement. The study finds applications in chemical reaction engineering processes, magnetic materials processing, solar collector energy systems, etc.

Chapter 5: In this chapter, finite element method with Hermite interpolation polynomials is employed to present the numerical solution for the mixed convective boundary layer, heat and mass transfer flow of a micropolar fluid through a vertical porous channel in the presence of transverse magnetic field and chemical reaction. Using Berman's similarity transformations, the governing boundary layer equations are transformed into a system of nonlinear ordinary differential equations of fourth order which are then solved numerically. The effects of the significant parameters namely cross-flow Reynolds number, Hartmann number, thermal buoyancy parameter, species buoyancy parameter, material parameter, chemical reaction parameter on the velocity, micro-rotation, temperature, concentration and also on the skin friction coefficient, local Nusselt number and Sherwood number are investigated. The study finds applications in transpiration cooling problems, gaseous diffusion and boundary-layer control.

Chapter 6: The steady, two-dimensional, heat and mass transfer of a mixed convection magneto-micropolar fluid flow over a non-permeable linearly stretching cylinder embedded in a porous medium in the presence of thermal radiation and first order chemical reaction with convective boundary condition is investigated in this chapter. Graphical variations of the velocity, micro-rotation, temperature and concentration functions across the boundary layer are presented to depict the influence of the controlling parameters. The study finds applications in chemical reaction engineering processes, magnetic materials processing, solar collector energy systems, etc.

Chapter 7: It is the final and concluding chapter. It summarizes the work of chapters 2-6 and discusses the future scope of the above studies. This chapter is followed by the bibliography.

LIST OF PUBLICATIONS

Publications in International Journals

1. Swapna G., Kumar L., Bég O.A., Singh B., "*Finite element analysis of radiative mixed convection magneto-micropolar flow in a Darcian porous medium with variable viscosity and convective surface condition*", Heat Transfer - Asian Research, vol. 44, pp. 515-532, 2015.

   H-Index: 14, **H5-Index:** 12, **Journal Charge:** No, **Peer Reviewed:** Yes, **Indexing:** Scopus, **Publisher:** John Wiley and Sons Inc. (United States), **Oral Presentation:** No.

2. Swapna G., Kumar L., Rana P., Singh B., "*Finite element modeling of a double-diffusive mixed convection flow of a chemically-reacting magneto-micropolar fluid with convective boundary condition*", Journal of the Taiwan Institute of Chemical Engineers, vol. 47, pp. 18-27, 2015.

   Impact Factor: 3.0, **H-Index:** 28, **H5-Index:** 34, **Journal Charge:** No, **Peer Reviewed:** Yes, **Indexing:** SCI, **Publisher:** Taiwan Institute of Chemical Engineers (Taiwan), **Oral Presentation:** No.

3. Swapna G., Kumar L., Bhardwaj N., "*Study of effects of radiation and magnetic field on the mixed convection micropolar fluid flow towards a stagnation point on a heated vertical permeable plate using finite element method*", International Journal of Mechanics and Systems Engineering, vol. 5, pp. 1-13, 2015.

   H-Index: 4, **Journal Charge:** No, **Peer Reviewed:** Yes, **Indexing:** Google Scholar, **Publisher:** American V-King Scientific Publishing (USA), **Oral Presentation:** No.

Publication in International Conference

1. Kumar L., Swapna G., Singh B., "*Finite difference solution of the mixed convection flow of MHD micropolar fluid past a moving surface with radiation effect*", F-and-B'11 Proceedings of the 4th WSEAS International Conference on Finite Differences - Finite Elements - Finite Volumes - Boundary Elements WSEAS (2011), pp. 41-46, April 28-30, 2011, Paris, France.

   H-Index: 2, **Journal Charge:** No, **Peer Reviewed:** Yes, **Indexing:** Scopus, **Publisher:** World Scientific and Engineering Academy and Society-WSEAS, (United States), **Oral Presentation:** No.

Communicated Papers in International Journals:

1. Swapna G., Kumar L., Bég O.A., Singh B., "*Finite element study of radiative mixed convection magneto-micropolar flow with a convective surface boundary condition*", communicated to Canadian Journal of Physics, July 2015.

   Impact Factor: 0.928, **H-Index:** 37, **H5-Index:** 18, **Journal Charge:** No, **Peer Reviewed:** Yes, **Indexing:** Scopus, SCI, **Publisher:** Faculty of Computers and Information, Cairo University (Canada), **Oral Presentation:** No.

2. Swapna G., Kumar L., Bég O.A., Singh B., "*Numerical study of mixed convective radiative magneto-micropolar transport in non-Darcy porous media with a convective surface boundary condition*" communicated to Journal of Porous Media, September 2013.

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3. Swapna G., Kumar L., Rana P., Singh B., "*Hermite FEM solution to MHD micropolar heat transfer characteristics in porous channel*", communicated to Journal of Engineering Thermophysics, Jan 2016.

   Impact Factor: 0.556, **H-Index:** 8, **H5-Index:** 10, **Journal Charge:** No, **Peer Reviewed:** Yes, **Indexing:** Scopus, SCIE, **Publisher:** Maik Nauka-Interperiodica Publishing (Russian Federation), **Oral Presentation:** No.

4. Swapna G., Kumar L., Rana P., Singh B., "*Finite element study of double-diffusive mixed convection magneto-micropolar flow in a porous medium with thermal radiation, chemical reaction and convective surface condition*", communicated to Alexendria Engineering Journal, June 2015.

   H-Index: 7, **H5-Index:** 14, **Journal Charge:** No, **Peer Reviewed:** Yes, **Indexing:** Chemical Abstracts, **Publisher:** Alexendria University (Egypt), **Oral Presentation:** No.

Papers Presented in National Conferences

1. Swapna G., Kumar L., Singh B., "*Effect of radiation on the stagnation point flow of MHD micropolar fluid towards a heated surface with suction*", International Conference on Advances in Modeling, Optimization and Computing (AMOC-2011), Book of abstract pp. 19, December 5-7, 2011, IIT, Roorkee, India.
2. Swapna G., Kumar L., Singh B., "*Finite element solution of the flow of a magneto-micropolar fluid past a continuously moving plate with a convective surface boundary condition*", National Conference on Contemporary Developments in Mathematical Sciences and Computing (CDMSC-2013), Book of abstract pp. 21, February 2-3, 2013, Galgotias University, Greater Noida, Uttar Pradesh, India.

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