A STUDY OF GROWTH PROPERTIES AND SPACES OF VECTOR VALUED DIRICHLET SERIES

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By

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SYNOPSIS

1. HISTORY

The abstract tendency in analysis which developed into what is today known as "Functional Analysis" began at the turn of 20^{th} century with the work of Volterra, Fredhlolm, Hilbert, Riesz and Banach .To develop the concept of distance in abstract linear spaces, the ideas of 'norm' was first formulated by F. Riesz in 1918. However, an abstract and full treatment to the subject was given by S.Banach in his book [4] which was published in 1932.This book was tremendously influential and signified the beginning of the systematic study on normed linear spaces. In last five decades, the research activities in this area grew tremendously. Sequences and in general functions of a real or complex variable were treated as elements of some abstract linear spaces. A suitable norm was defined on these linear abstract spaces to make them a Normed Linear Space (NLS), in which ideas of convergence and continuity have been developed. A complete NLS is known as a Banach space [52] which became a foundation stone for today's analysis.

As a result Banach space theory gained very much depth as well as scope in many branches of mathematical sciences and engineering. However, not enough work seems to have been done dealing with the inter-play between functional analysis and the theory of analytic functions of a complex variable, the reason might be that the functional analysis techniques are essentially of real variable character. But there are parts of the theory of analytic functions which blend beautifully with the concept and methods of functional analysis which lend clarity and elegance to the proof of classical theorems and thereby making the results available on more general settings. A testimony of these facts are the books of Hille and Phillips [14], Taylor [48], Hoffman [15], Willnasky [50, 51], Porcelli [33], Duren [12], Maddox [30], Rudin [38, 39], Limaye [28], Kreyszig [25], Wojtaszczyk [52], Schechter [40] and Lax [27] etc.

In the theory of functions of a complex variable, those functions which are representable in the form of a power series or a Dirichlet series play an important role. Many problems of science and engineering modelled as Differential or Integral equations have solutions in the form of a power series or a Dirichlet series for the solution function analytic in a domain. So the class of such functions needs to be treated separately. Many mathematicians deal with such type of classes of functions and study their properties as a normed linear spaces (NLS) or Banach spaces, ours is another attempt in this direction. In this work we have studied the various spaces of functions representable in the form of a Dirichlet series and obtain their important properties.

1.1 Dirichlet Series.

A series of the form,

$$F(s)=\sum_{n=1}^{\infty} f(n)n^{-s} ,$$

where the variable s may be complex or real and $f(n)$ is a number-theoretic function, is called the original Dirichlet series. The sum $F(s)$ of the series is called the generating function of $f(n)$ .The Riemann zeta-function

$$\xi(s)=\sum_{n=1}^{\infty} n^{-s} =\prod_{p}\left(1-p^{-s}\right)^{-1},$$

where n runs through all positive integers and p runs through all primes is the special case where $f(n)=1$ identically. It is fundamental to the study of prime numbers and many generating functions are combinations of this function.

In 1988 Carlos A. Berenstein and Daniele C. Struppa [8], defined Dirichlet series as a very particular case of solutions of homogeneous convolution equations, and considered these objects in the framework of a general theory of Fourier analysis on non-algebraic varieties. Their study provided a wealth of results on solutions of systems of linear partial differential equations with constant coefficients.

The above series was later generalized to the form

$$\sum_{n=1}^{\infty} a_n e^{s\lambda_n} \text{ , } s=\sigma+it, \text{ (}\sigma \text{ and } t \text{ real variables)},$$

where the sequence $\{a_n\}\subseteq C$ (the field of complex numbers) and $\{ \lambda_n \}$ is a strictly increasing sequence of positive numbers , i.e.

$$0\le\lambda_1<\lambda_2<\lambda_3...<\lambda_n\to\infty,\ \ (n\to\infty).$$

Dirichlet and Dedekind considered only real values of the variable 's' and obtained many important results. Cahen [7] obtained the first result involving the complex values of 's' which determined the nature of the region of convergence of the Dirichlet series. Later, Littlewood [29] succeeded in showing that the Dirichlet series could be useful in the study of entire functions, while Easterman [13] used Dirichlet series in the study of meromorphic functions. The growth of sum function $f(s)$ of the Dirichlet series seems to have been first studied by Doetsch [11]. However, a systematic study of the growth of $f(s)$, when $f(s)$ is an entire function was made by Ritt [37], who in fact considered the Dirichlet series with positive exponents instead of negative ones, in order to have an analogy with the study of the growth of entire functions represented by Taylor series. Significant contributions were made by Sugimura [46], Izumi [17] and Mandelbrojt [31]. But a vast enrichment to this field with new and fruitful ideas came in the wake of the works of Tanaka [47], Azpeitia [1, 2, 3,], Rahman [34, 36] and Dagene [9].

Consider the Dirichlet series

$$\sum_{n=1}^{\infty} a_n e^{s\lambda_n}, \; s = \sigma + it \; (\sigma, t, \text{ real variables}) \tag{1}$$

where $\{a_n\}$ is a sequence of complex numbers and $\{\lambda_n\}$ satisfies

$$0 \le \lambda_1 < \lambda_2 < \lambda_3 \ldots < \lambda_n \to \infty, \;\; (n \to \infty), \tag{2}$$

It is known [31] that the series (1) converges in a left half plane and that the sum function $f(s)$ of the series (1) is holomorphic in its half plane of convergence.

If σ_c and σ_a denote, respectively the abscissa of convergence and the abscissa of absolute convergence of the series (1) then it follows that (see [31])

$$\sigma_c = -\limsup_{n\to\infty} \frac{\ln |a_n|}{\lambda_n}$$

and

$$0 \le \sigma_c - \sigma_a \le \limsup_{n\to\infty} \frac{\ln n}{\lambda_n} \equiv D \tag{3}$$

Thus if $D<\infty$ and $\sigma_c=\infty$, $f(s)$ represents an entire function and by (3), $\sigma_a=\infty$ so that the series (1) converges absolutely at every point of the finite complex plane. Further if $D=0$, then it is clear from the above that

$$\sigma_c=\sigma_a=-\limsup_{n\to\infty}\frac{\ln|a_n|}{\lambda_n}. \tag{4}$$

1.2 Vector Valued Dirichlet Series.

In 1983, B.L.Srivastava [43] introduced a new class of Dirichlet series which is called vector valued Dirichlet series. He modified the above Dirichlet series by considering the sequence $\{a_n\}$ as a member of a complex Banach space. Subsequently B.L.Srivastava also studied growth properties of analytic functions represented by vector valued Dirichlet series and obtained the coefficient characterization of their order and type.

Consider the VVDS,

$$f(s)=\sum_{n=1}^{\infty}a_n e^{s\lambda_n} \tag{5}$$

where $\{a_n\}\subseteq E$ and $\lambda_n^{'s}\in\Box$ satisfy the conditions

$$0\le\lambda_1<\lambda_2<\lambda_3\ldots<\lambda_n\to\infty,\ (n\to\infty), \tag{6}$$

$$\limsup_{n\to\infty}\frac{n}{\lambda_n}=D'<\infty. \tag{7}$$

If σ_c and σ_a denote the abscissa of convergence and abscissa of absolute convergence of the series and the exponents $\{\lambda_n\}$ satisfy the condition

$$\limsup_{n\to\infty}\frac{\ln n}{\lambda_n}=D=0 \tag{8}$$

then, we have, $$\sigma_c=\sigma_a=-\limsup_{n\to\infty}\frac{\ln\|a_n\|}{\lambda_n}=R(\text{say}). \tag{9}$$

Srivastava ([43], Theorem 2.2 C) showed that the series (5) represents a vector valued holomorphic function in the half plane $\sigma<R$, $(-\infty<R<\infty)$ and is called vector valued analytic Dirichlet series. If $\sigma_c=\sigma_a=\infty$ then the series represents a vector-valued entire function and series is called vector valued entire Dirichlet series.

1.3 Growth Properties of Vector Valued Dirichlet Series.

Let $f(s)$ be a vector valued entire function; its maximum modulus is defined as

$$M(\sigma, f) = M(\sigma) = \operatorname*{lub}_{-\infty<t<\infty} \| f(\sigma+it) \|. \tag{10}$$

Then following Doetsch [10], it can be shown that $\ln M(\sigma)$ is an increasing convex function of σ. The growth parameters order ρ and lower order λ of $f(s)$ were introduced by Srivastava [43] as

$$\begin{matrix}\rho \\ \lambda\end{matrix} = \lim_{\sigma\to\infty} \begin{matrix}\sup \\ \inf\end{matrix} \frac{\ln\ln M(\sigma)}{\sigma}, \quad 0 \le \lambda \le \rho \le \infty.$$

If $(0 < \rho < \infty)$, then type T and lower type τ of $f(s)$ are defined as

$$\begin{matrix}T \\ \tau\end{matrix} = \lim_{\sigma\to\infty} \begin{matrix}\sup \\ \inf\end{matrix} \frac{\ln M(\sigma)}{e^{\sigma\rho}}, \quad 0 \le \tau \le T \le \infty.$$

The vector valued entire function f is said to be of regular growth if $\lambda = \rho$ otherwise is of irregular growth. The function f is said to be of perfectly regular growth if $0 < \tau = T < \infty$. The vector valued entire function f is said to have growth $\{\rho, T\}$ if its order does not exceed ρ and its type does not exceed T if it is of order ρ.

Let $\sum_R$ denote the class of all functions $f(s)$ defined by (5) representing a vector valued holomorphic function in the half plane $\sigma < R$ $(-\infty < R < \infty)$. Then the order ρ and lower order λ of $f(s)$ are given by [43]

$$\begin{matrix}\rho \\ \lambda\end{matrix} = \lim_{\sigma\to R} \begin{matrix}\sup \\ \inf\end{matrix} \frac{\ln\ln M(\sigma)}{-\ln(1-e^{\sigma-R})}, \quad 0 \le \lambda \le \rho \le \infty.$$

For $(0 < \rho < \infty)$, the type T and lower type τ are defined as

$$\begin{matrix}T \\ \tau\end{matrix} = \lim_{\sigma\to R} \begin{matrix}\sup \\ \inf\end{matrix} \frac{\ln M(\sigma)}{\{1-e^{\sigma-R}\}^{-\rho}}, \quad 0 \le \tau \le T \le \infty.$$

2. OBJECTIVES OF THE STUDY.

The main objectives of the work reported in the thesis are:

- to study various properties of the spaces of functions represented by vector valued Dirichlet series,

- to obtain the properties of a sequence space of the coefficients of vector valued Dirichlet series,
- to obtain some coefficient multipliers for some classes of vector valued Dirichlet series,
- to obtain characterization of continuous linear transformations of spaces,
- to obtain the nature of the dual spaces of the sequence space and coefficient multipliers for some classes of vector valued Dirichlet series with several variables,
- to study the growth properties of holomorphic vector valued Dirichlet series with complex exponents,
- to obtain the characterizations of order and type of vector valued Dirichlet series with complex exponents,
- to obtain the characterizations of order and the type of the analytic function $f(s)$ represented by a vector valued Dirichlet series with complex exponents in terms of the rate of decay of the approximation error.

3. THESIS OUTLINE

CHAPTER- 1
(INTRODUCTION)

First chapter of the thesis contains a brief history of Dirichlet series, vector valued Dirichlet series, some definitions and results which have been used in subsequent chapters. Further, a brief account of our work carried out in next chapters is also given.

CHAPTER- 2

SPACES OF FUNCTIONS REPRESENTED BY VECTOR VALUED DIRICHLET SERIES IN A HALF PLANE

Dirichlet series with real exponents which represent entire functions on the complex plane C have been investigated by many authors. Kamthan and Hussain [19, 21] have studied various properties of the spaces of entire functions defined by Dirichlet series. Several properties such as topological structures, linear continuous functionals, bases, etc., have been considered. Le Hai Khoi [23] obtained some results with Dirichlet series having negative real exponents which represent holomorphic functions in a half plane. In 1998, Arvind Kumar and

G.S.Srivatava [26] obtained the properties for a space of analytic functions f represented by Dirichlet series under which they become a Frechet space. Later, Archana and G.S.Srivastava [44] obtained these properties for a space of entire functions defined by vector valued Dirichlet series. The above properties have been discussed by considering the characterization of type.

In this chapter we have considered the space of analytic functions represented by vector valued Dirichlet series with positive exponents. We have obtained some properties of holomorphic Dirichlet series having positive exponents, whose coefficients belong to a Banach algebra.

We consider E to be a Banach algebra and describe a new space E_R .Firstly, we have studied the properties of a sequence space of the coefficients of Dirichlet series. Here, we follow the terminology used by Khoi in [23]. Next, we have introduced various dualities of the sequence space and study about coefficient multipliers between the spaces E_R and $l^p (0 < p \leq \infty)$.

In the next section of this chapter we have obtained various useful properties of the space of analytic functions represented by VVDS by considering the characterization of their order. We have also characterized the form of linear continuous functional on this space.

CHAPTER- 3

COEFFICIENT MULTIPLIERS ON SPACES OF VECTOR VALUED ENTIRE DIRICHLET SERIES

Le Hai Khoi studied about the spaces of analytic functions represented by Dirichlet series with negative exponents and obtained various results. In a series of papers [22-24] he has studied the convolution properties and multipliers for the Dirichlet series in the complex plane. In [23] he introduced various concepts of duality for sequence spaces. In this chapter we have studied a sequence space which depends upon the order and type of an entire function represented by vector valued Dirichlet series respectively and obtained the dual nature of these sequence spaces. We have also obtained some coefficient multipliers for some classes of vector valued Dirichlet series.

CHAPTER- 4

GROWTH PROPERTIES OF VECTOR VALUED DIRICHLET SERIES WITH COMPLEX EXPONENTS

The concept of generalized Dirichlet series was initiated by Andre Boivin and Changzhong Zhu [5].They obtained the growth properties of entire functions represented by these series. Later, Wen Ping Huang, Ju Hong Ning and Jin Tu [16] in 2009 made independent studies on these series. To the best of our knowledge, the growth of analytic functions represented by generalized Dirichlet series have not been studied so far. In this chapter we have introduced vector valued generalized Dirichlet series and obtained various properties of spaces of analytic function represented by this series. After that we have studied the growth properties of the derivative of function represented by vector valued Dirichlet series with complex exponents. We have obtained abscissa of convergence and characterizations of growth parameters of this series.

Let $\Lambda = \{\lambda_n = |\lambda_n| e^{i\,\alpha_n}; n = 1,2,3....\}$ be a sequence of complex numbers in the right half plane satisfying the following conditions:

$$\left.\begin{aligned} &\liminf_{n\to\infty}\left(|\lambda_{n+1}| - |\lambda_n|\right) = \delta(A) > 0; \\ &\sup\left\{|\arg\alpha_n| : n = 1,2,...\right\} \le \alpha < \frac{\pi}{2}; \\ &\limsup_{n\to\infty} \frac{n}{|\lambda_n|} = D < \infty. \end{aligned}\right\} \tag{11}$$

Let us assume that f is a function represented by vector valued generalized Dirichlet series

$$f(s) = \sum_{n=1}^{\infty} a_n e^{s\lambda_n};\ s = \sigma + it; \sigma \text{ and } t \text{ are real numbers,} \tag{12}$$

where $\{a_n\}$ is a sequence from the complex Banach algebra E and the sequence $\{\lambda_n\}$ is defined as above satisfying the conditions in (12).

Bovin and Zhu [5] obtained the growth properties of entire functions represented by Dirichlet series with complex exponents. Later, Ping et.al.[16] also obtained these characterizations. In 1998, Le hai khoi [23] obtained convergence properties of analytic functions represented by Dirichlet series. In this chapter we first obtain these properties for a space of analytic functions defined by vector valued generalized Dirichlet series. Later, we have introduced the growth parameters and obtained coefficient characterizations for order and type of analytic function represented by this series.

Derivative of VVDS with Complex Exponents.

In 1975, B.V.Vinnitskii [49] studied the properties of derivative of an entire function represented by Dirichlet series. Later, M. N. Sheremeta [41] obtained some growth characteristics of derivative of an entire function represented by Dirichlet series. Further he continued his work with S. I. Fedynyak [42] and obtained various growth properties of an analytic function and an entire function represented by Dirichlet series.

Let f be a function represented by vector valued generalized Dirichlet series (12) and analytic in the domain $G_0 = \{|\arg s| \leq \theta_0 < \pi/2; s = \sigma + it, \sigma, t \in \mathbb{R}\}$.

We have shown that the condition for convergence of the vector valued Dirichlet series given by (12) and its derivative is same. Further we can represent its derivative as

$$f'(s) = \sum_{n=1}^{\infty} a_n \lambda_n e^{s\lambda_n} \tag{13}$$

where $\{a_n\}$ is a sequence from the complex Banach algebra E and the sequence $\{\lambda_n\}$ is same as defined above satisfying the conditions in (11).

Later, we proved that the coefficient characterizations of order of functions represented by (12) and (13) are same. Similarly it is true for type also.

CHAPTER- 5

APPROXIMATION OF AN ANALYTIC FUNCTION REPRESENTED BY VECTOR VALUED GENERALIZED DIRICHLET SERIES

The coefficient characterization of order and type of analytic functions represented by Dirichlet series was given by O.P.Juneja and Krishna Nandan [18]. Later on, A.Nautiyal and D.P.Shukla [32] have characterized order and type of analytic functions represented by Dirichlet series in terms of rate of decay of approximation error. To the best of our knowledge, the characterization of order and type of functions represented by vector valued Dirichlet series in terms of rate of decay of approximation error has not studied so far.

In the previous chapter, we have introduced the concept of growth of analytic functions represented by vector valued Dirichlet series with complex exponents. In these series, we have taken the coefficients from a complex Banach algebra. In this chapter, we have introduced the approximation error of these series with respect to a class of exponential polynomials. We have characterized the order and the type of the analytic function $f(s)$ represented by a vector valued Dirichlet series with complex exponents in terms of the rate of decay of the approximation error introduced. Our results generalize some of the earlier results obtained by A.Nautiyal and D.P.Shukla [32] for classical Dirichlet series.

CHAPTER-6

COEFFICIENT MULTIPLIERS ON SPACES OF ENTIRE FUNCTIONS REPRESENED BY VECTOR VALUED DIRICHLET SERIES WITH TWO VARIABLES

The study of Dirichlet series in two complex variables was initiated by S.K.Bose and D.Sharma [6]. Later, S. Daoud [10] made a study on spaces of functions represented by Dirichlet series in two complex variables. She obtained various useful properties related to entire functions represented by Dirichlet series in two complex variables. Very recently, in 2011, Srivastava and Sharma [45] introduced the vector valued Dirichlet series with several complex variables.

Let X denote the space of all entire functions defined by vector valued Dirichlet series of two complex variables s_1 and s_2 such that when $f \in X$,

$$f(s_1,s_2)=\sum_{m,n=1} a_{mn}e^{(\lambda_m s_1+\mu_n s_2)} \quad ,(s_j=\sigma_j+it_j, j=1,2) \tag{14}$$

where a_{mn}'s belong to a complex Banach algebra E with the unit element ω; λ_m's and μ_n 's are increasing sequences such that $0<\lambda_1<\lambda_2<\lambda_3 ....<\lambda_m ......;\ \lim_{m\to\infty}\lambda_m=\infty$ and $0<\mu_1<\mu_2<\mu_3 ....<\mu_n ......;\ \lim_{n\to\infty}\mu_n=\infty$ and

$$\limsup_{m,n\to\infty}\frac{\ln(m+n)}{\lambda_m+\mu_n}=0, \tag{15}$$

Such a series is called a vector valued Dirichlet series in two complex variables. Further, Srivastava and Sharma obtained the coefficient characterizations of order and type of vector valued Dirichlet series in two variables.

Le Hai Khoi was the first author to introduce the concept of the coefficient multipliers on spaces of functions represented by Dirichlet series. The coefficient multipliers on spaces of functions represented by vector valued Dirichlet series of several complex variables have not been studied yet.

In this chapter we have extended the results to the entire functions represented by vector valued Dirichlet series of several complex variables. Firstly, we have studied about a sequence space which depends upon the order of an entire function represented by vector valued Dirichlet series of several complex variables and obtained the dual nature of this sequence space. In the later part the coefficient multipliers for some classes of vector valued Dirichlet series of several complex variables have been obtained. For the sake of simplicity, we have considered the functions of two variables only. These results can easily be extended to functions of several complex variables.

Secondly, we have studied about a class of sequence spaces defined by using the type of an entire function represented by vector valued Dirichlet series of two complex variables. The main results of this section concern with obtaining the nature of the dual spaces of this sequence space and coefficient multipliers for some classes of vector valued Dirichlet series.

FUTURE SCOPE

The study of growth of Dirichlet series with complex exponents is a relatively new topic and has vast scope for further study. It is known that the characterizations of growth parameters are obtained as estimates and not as equality relations. It will be interesting to find the conditions under which various inequalities will turn to equality. One could also develop the study of mean value functions and their growth. The existence of proximate orders and proximate type for Dirichlet series with complex exponents can also be explored. We could also develop a new study by using random Dirichlet series with complex exponents and define growth parameters. By using those growth parameters, we can construct various linear spaces of random Dirichlet series and endow them with topological structures. The topological properties of these spaces can also be obtained.

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