Computational Thinking in Early India and its Current Relevance

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CS

Two fundamental aspects

- create new *computing algorithms* and *machines* that have powerful *computational and cognitive abilities*: this includes development of new techniques of **representing and manipulating knowledge, inference and deduction.**
- develop techniques that make the elucidation of the *computational structur*e of **nature** and the **mind** easier.

Computational thinking in early India?

- Grammarians
  - Grammar ~ computation!
- Logicians
  - Logic ~ computation!
- Connection with “cognitive science”?
  - “inner sciences” like Yoga wrt mind sciences (psychology, neuro)
- Qn of significance to Indians as well as cognitive and computer scientists: Can early Indian logic and grammar give us some insights that may be of use in making further advance in cognitive and computer sciences? [~ Bhate & Kak]

The Two A's of Computational Thinking (C.T.) (from J. Wing'06)

- Abstraction
  - C.T. operates in terms of multiple layers of abstraction simultaneously
  - C.T. defines the relationships the between layers
- Automation
  - C.T. thinking: mechanizing abstraction layers and their relationships
  - Mechanization: due to precise & exacting notations and models
  - some "machine" below (human or computer, virtual or physical)
- Computational Thinking
  - Abstraction, Mechanization, Recursion & Bootstrapping
- *They give us the ability and audacity to scale*
- Interestingly, many of these handled in our tradition

Axiomatic Derivation vs Computation

- (old) AI vs (new) ML? Deduction vs computation?
- Deductive/ “Logical” vs Computational Positivism (R. Narasimha)
- Positivism: facts are the only possible objects of knowledge and science the only valid knowledge. No metaphysics!
  - “logical” positivism of the famous Vienna Circle (scientists, mathematicians and philosophers) in first half of 20th c.
  - central tenet verifiability - a statement that cannot be verified automatically held to be meaningless. (Godel/Popper sank this school)
  - only two types of meaningful statement: the necessary truths of logic, mathematics and language, and empirical propositions about the rest of the world.
    - Wittgenstein: propositions of logic and mathematics are tautologies!
- Most impressive achievement in India: Paanini's grammar
    - Describe a living language “completely”
    - Minute details handled (further refined by Patanjali, Kaatyaayana)
    - Is this an example of Computational Positivism?

Patanjali

- Language innate (a view popularized by Chomsky also much much later)
- Argues that simpler to enumerate the correct expressions of the language than to enumerate the incorrect expressions.
- How is the enumeration is to be done?
  - By listing them? No, that would be difficult.
  - Brhaspati (the teacher of the gods) taught Indra a work containing all correct expressions of Sanskrit for a thousand divine years (360,000 human years), and still did not come to an end!

Hence Generative linguistics!

- First and most sophisticated form in Paanini

Panini's “expert system”

पाणिनीय पञ्चग्रन्थी

- Code (सूत्र):
  - अष्टाध्यायी सूत्रपाठः
  - Rule-based
  - Mostly “declarative” (stateless) but some places “imperative” (stateful)
  - लिङ्गानुशासनम् determines the gender of linguistic items, based on their structure and meaning
  - पाणिनीयशिक्षा
- Data (input)
  - धातुपाठः
  - गणपाठः
- Data (output)
  - पद
  - Gender
  - accent
- Control
  - “Blackboard” arch

Term rewriting systems

- Post production systems studied mathematically in 20's
  - Shown to be equivalent to Turing machines
- Chomsky hierarchy (add terminals/non-)
  - Regular grammars: *A=>aB or A=>a* (FSMs)
  - Context free grammars: *A=>string* (NDPDA)
  - Context sensitive grammars: *x A y=>x string y*
    - (Linear Bounded nondet automaton)
  - Unrestricted grammars: *string => string* (Turing Ms)
- *Panini?*
  - *Elements of CSGs + Post production systems*
  - *With interesting technical devices for termination, rule inheritance (anuvritti)/grouping*
  - *With context sensitive application of rules (exceptions)*

Panini Sutra forms

X⇒Y(Z)

- ⇒ stands for is or becomes, and ( ) stands for when
- X stands for the subject
- Y represents predicate
- Z stands for environment

For substitute: B (gen) ⇒ C (nom) (D(loc))

१.१.४९ षष्ठी स्थानेयोगा । (6th)

For suffix: A (abl) ⇒ C (nom) (D(loc))

१.१.६६ तस्मिन्निति निर्दिष्टे पूर्वस्य । (7th) eg. ६.१.७७ इकः(6) यण् (1) अचि (7)।

१.१.६७ तस्मादित्युत्तरस्य । (5th) eg. ८.१.२८ तिङ् अतिङः(5)। (a तिङ् after a अतिङ् becomes anudaatta)

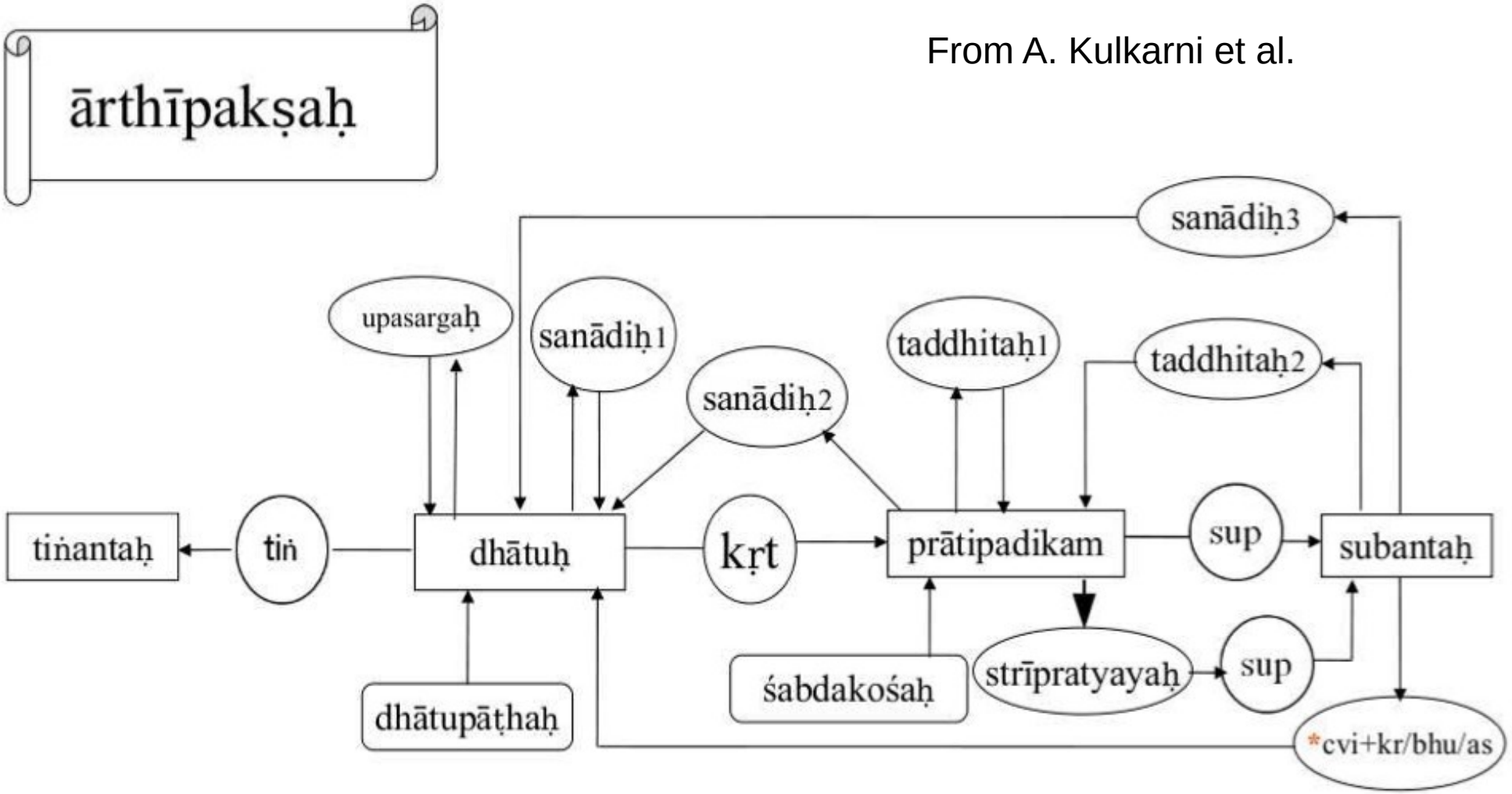
More general: A[B=>C]D

- A: abl (5)
- B: gen (6)
- C: nom (1)
- D: loc (7)

Setting इत् bit

```
इत् = false;
if (उपदेश) {
   if (अच् अनुनासिक) इत् = true;
   if (हल् अन्त्यम्) इत् = true;
        if (विभक्ति at end) इत् = false;
        if (तु, स्, म् at end) इत् = false;
   if (ञि-टु-डु at beginning) इत् = true;
   if (षः प्रत्यय) इत् = true;
   if (चु or टू) इत् = true;
   if (अतद्धित)
      if (ल-श-कु) इत् = true;
}
```

From A. Kulkarni et al.



	Legends:
tiṅantaḥ: verbal form	**tiṅ**: verbal suffix
prātipadikam: nominal base	**sup**: nominal suffix
kṛt: derivational suffix	**sanādiḥ**: derivational suffixes
upasargaḥ: verbal prefix	**śabdakoṣaḥ**: list of nominal bases
strīpratyayaḥ: feminine suffixes	**taddhitaḥ**: nominal derivational suffixes
subantaḥ: nominal form	**dhātupāṭaḥ**: list of verbal roots

Computational Positivism

- computation and observation, when in agreement, constitute the only form of valid knowledge; models, logic, metaphysics etc. are either secondary or not relevant.
  - Models may *not* be unique (in the sense that different models may yield very similar results in a domain of interest) and logic tautology (Wittgenstein)!
- This attitude, often implicitly and occasionally explicitly, informed the classical Indian mathematical approach to astronomy
- Also, seen in
  - Infinite series in trigonometry (Kerala school of mathematics)
    - Reasoning on the algebraic form of term that will converge better
  - Theory of induction (Udayana/Gangesha)
    - Restrict it to be meaningful and useful
  - Theory of universals (Udayana)
    - Avoid “naive set” theory difficulties with jaatibaadhakaas
  - Panini an inspiration?
- Meet the problem headon!

Astronomy

- Same physical system for all but diff approaches in diff civilizations
  - Some complexity: planets can exhibit retrograde motion
- Model based (Ptolemy) vs “Model-free” (Aryabhata)
- Ptolemy: (Greeks) circle a perfect figure: planets tend to move in circles but what about retrograde motion?
  - Model extensions to the rescue: epicycles: circles whose centres move on other circles
  - Adds complexity to model to fit data
  - Almagest first few chaps only about model
    - Then deduction!??
  - Surprise (?) Many predictions whose accuracy that would not be improved in Europe till some time after Newton

Aryabhatiyam

- Epicycle only used as a convenient representation of motion
  - no mention of a physical or kinematic model, therefore no justification
  - no notion that the circle is a perfect figure, etc.
- *Aryabhatiyam* best seen as a collection of 50+ algorithms
  - Starts with ingenious system for expressing numbers
  - Lists of various numerical parameters required later
  - Next physical concepts: eclipses caused by shadows, significance of relative motion, and proposes that the earth spins on its axis (therefore not stationary as in Ptolemy's model)
- Provides short, effective methods of calculation rather than a basic model from which *everything* can be deduced
  - Trigonometry, Equations
- Describes algorithmic or computational astronomy!
- Non-model based: handle actual reality rather than ideologically inspired/imposed models
    - Aaryabhata: heliocentric model
    - Niilakantha: geoheliocentric model

Kerala School

- Group of astronomers and mathematicians who, over a period of some three centuries, produced some very innovative and powerful mathematics applied to astronomy
- Goal: *drg.ganitaikya,* the identity of the seen and the computed
- Effort to find “best” algorithms or computational procedures that made the best predictions as determined by comparison with observation.
  - over a period of time, discrepancies between computation and observation tended to increase
- Nilakantha (1444- 1545 CE): “logical reasoning is of little substance, and often indecisive”
  - Opposite of Euclidean method of going from well stated axioms through a process of purely logical deduction to theorems or conclusions
  - Explicit statements such as: the best mathematicians have to sit together and decide how the algorithms have to be modified or revised to bring computation back into agreement with observations!
- Experimental Mathematics; Constructive Mathematics; Undecidable propositions

- Symbolic
  - Number systems: recursion
  - Sanskrit grammar: term-rewriting
- Numeric
  - Computational: astronomy
  - Combinatoric: music
- Logical
  - Udayana/Gangesha:
    - Theory of universals
    - Theory of induction
  - Argumentation

Recursion

- Number systems using positional notation 1st good example of use of recursion
- एदश... = ए + द*10 + श*100... =

  ए + 10*(द + 10*(श+...))
- एदश... + ए'द'श'... = (ए+ए')(द+द')(श+श')...
  - Addition natural!
- ...abcd = (...(a*10+b)*10+c)*10+d
  - Addition “reverse direction”
- (अङ्कानाम् वामतो गतिः)

तैत्तरीय U. happiness levels!

- मनुष्य 1
  - Healthy, young man!
- मनुष्यगंधर्व 10^2
- देवगंधर्व 10^4
- पितृ 10^6
- अजानजदेव 10^8
- कर्मदेव 10^10
- देव 10^12
- इन्द्र 10^14
- बृहस्पति 10^16
- प्रजापति 10^18
- (2^64= 92233720368547758O8)
  - 9.2E18

For a 64b machine, OVERFLOW here!

- ब्रह्म 10^20

Algorithmic Thinking

Considerable tradition in India in ``algorithmic'' thinking, so can see many elements of an informal ``programming'' style written in cryptic Sanskrit. Examples:

- Multiplication/Division/Sq root
- Combinatorics
- Number theory: Kuttaka/Chakravala
- Infinite series: calculate pi
- Coding for error correction and self-description
- Backtracking? Solutions of a knight's tour on 4x8 board given by Vedanta Desikan (13th c.) in ``Paaduka Sahasram''

chitrakavya

- Backtracking
  - 4x8 knights tour

(from “The wonder that is Sanskrit” Vijay/Sampad)

स्थि	रा	ग	सां	स	दा	रा	ध्या
1	30	9	20	3	24	11	26
वि	ह	ता	क	त	ता	म	ता
16	19	2	29	10	27	4	23
स	त्पा	दु	के	स	रा	सा	मा
31	8	17	14	21	6	25	12
रं	ग	रा	ज	प	द	न्न	य
18	15	32	7	28	13	22	5

स्थिरागसां सदाराध्या विहताकततामता ।
सत्पादुके सरासा मा रङ्गराजपदं नय ॥[21]

O sacred sandals (*pādukā*) of the *Brahman*, you are always adored by those who have committed unpardonable sins; you remove all that is sorrowful and unwanted; you create a musical sound; (be pleased) and lead me to the feet of Lord Rangaraja (Rama).

When this verse is read as per the movement of a knight on the chessboard, it creates the following *śloka*.

स्थिता समयराजत्पागतरा मादके गवि ।
दुरंहसां सन्नतादा साध्यातापकरासरा ॥

The sandals (*pādukā*) which protect those who shine by their right attitude, whose place is in the centre of the blissful rays, which destroy the melancholy of the distressed, whose radiance brings peace to those who take refuge in them, which move everywhere, – may those golden and radiating sandals of the Brahman lead me to the feet of Lord Rangaraja.

- “From the system of nine heavens [marked] with nine figures [God] cast the Indian numerals onto the earth board”
  - Nizami(?-1203 CE) Persia
- **paati-ganita** (arithmetic; “board-computation” for the dust board, or sandbox: perms/combinations, quadratic eqns, summing series) *vs*
- **biija-ganita** (algebra; “seed-computations” for the manipulation of equations involving an unknown or bija: solving eqns up to 2nd order in 1+ unknowns, and indeterminate eqns of 1st and 2nd degree.)
  - (from Ency. Brit.)

Multiplication in India (paati-ganita)

	3	2	5
2	8		

6	3	2	5
2	8		

8	4	2	5
2	8		

8	4	2	5
	2	8	

8	8	2	5
	2	8	

8	9	6	5
	2	8	

8	9	6	5
		2	8

- Align “lsb” of one (B:b_1b_2) with the “msb” of the other (A:$a_1a_2a_3$)): b_2 with a_1
- Multiply B and a_1
  - Can rub a_1 out!
  - Carry? Extend to left & add!
- Shift B right
- Multiply B and a_2
  - Carry? Add to prev existing
  - a_2 also gone
- Shift B right
- Multiply B and a_3
  - a_3 also gone but now result avlbl

Looks like an alg on a Turing Machine!

Division/Sqroot

- Division considered simple and just assumed by Aaryabhata and other great mathematicians of that time
  - In Europe division alg only about 10 centuries later based on the Indian methods
- Square roots use a “division” like method
  - Odd places: varga, even places: avarga
  - Cube roots too!

GAṆITAPĀDA, VERSE 4

भागं हरेदवर्गान्नित्यं द्विगुणेन वर्गमूलेन ।
वर्गाद्वर्गे शुद्धे लब्धं स्थानान्तरे मूलम् ॥ ४ ॥

Always divide the non-square (even) place by twice the square root [already found]. Having subtracted the square [of the quotient] from the square (odd) place, the quotient gives the [digit in the] next place in the square root

Translation [2]: (Having subtracted the greatest possible square from the last odd place and then having written down the square root of the number subtracted in the line of the square root) always divide the even place (standing on the right) by twice the square root. Then, having subtracted the square (of the quotient) from the odd place (standing on the right), set down the quotient at the next place (i.e., on the right of the number already written in the line of the square root). This is the square root. (Repeat the process if there are still digits on the right).

$$
\begin{array}{l}
\overline{1}\ \hat{1}\ \overline{9}\ \hat{4}\ \overline{3}\ \hat{9}\ \overline{3}\ \hat{6} \\
-\ 9 \\
\hline
\quad 2\ 9 \\
-\ 2\ 4 \\
\hline
\quad\ \ 5\ 4 \\
-\ 1\ 6 \\
\hline
\quad\ \ 3\ 8\ 3 \\
-\ 3\ 4\ 0 \\
\hline
\quad\ \ \ 4\ 3\ 9 \\
-\quad 2\ 5 \\
\hline
\quad\ \ \ 4\ 1\ 4\ 3 \\
-\ 4\ 1\ 4\ 0 \\
\hline
\quad\ \ \ \ \ \ 3\ 6 \\
-\ 3\ 6 \\
\hline
\quad\ \ \ \ \ \ \ \ 0
\end{array}
$$

Root result: 3 4 5 6

1. Pick the digit d_i such that $\frac{i}{2}$ = integer,
   Where, $\frac{i+k}{2} \neq$ integer and $\frac{i}{2} > \frac{i-k}{2}$, k =1,2,3, … .
2. $p = \left\lfloor \frac{n-1}{2} \right\rfloor$.
3. If d_{i+1} exists then $a = 10 \times d_{i+1} + d_i$, else $a = d_i$.
4. Choose A such that $A^2 \leq a$ and $a - A^2$ is minimum.
5. $S = a - A^2$.
6. $R = A$.
7. $y = 10 \times S + d_{i-1}$.
8. $S = y \bmod (2 \cdot R)$.
9. $B = \left\lfloor \frac{y}{2 \cdot R} \right\rfloor$.
10. $R = 10 \times R + B$.
11. $c = 10 \times S + d_{i-2}$.
12. $S = c - B^2$.
13. $i = i - 2$.
14. $p = p - 1$.
15. If $p \neq 0$ then go to 7, else quit.

(Aryabhata's Root Extraction Methods
Abhishek Parakh)

Combinations

- Susruta (Charaka Samhita) 2000+ years before
  - Medicine: sweet, sour, salty, peppery, bitter or astringent
  - Mix
    - any 2 qualities: 15 possibilities (${}^6C_2$)
    - any 3 qualities: 20 possibilities (${}^6C_3$)
    - any 4 qualities: 15 possibilities (${}^6C_4$)
    - any 5 qualities: 6 possibilities (${}^6C_5$)
    - any 6 qualities: 1 possibility (${}^6C_6$)
- Bhaskara (c.1150) (Lilavati)
    - ${}^nC_k$ formula: (n(n-1)...(n-k+1))/(k(k-1)...1)
    - Multinomial Theorem

Combinatorics (Pingala's/Gopala's #s)

- Gopala (< 1135 CE) and Hemachandra (c. 1150): find number of ताला (rhythmic patterns) for *n* मात्रा (beats) (F(*n*)) with anudruta (1-beat) and druta (2-beat): (total time fixed)
  - $\text{ताला}(\text{म}) = \text{ताला}(\text{म}-1) + \text{ताला}(\text{म}-2)$, or F(*n*) = F(*n*-1) + F(*n*-2)
    - Fix one anudruta as the 1st part. Remaining (*n*-1) beats have F(*n*-1) distinct possibilities
    - Next, fix one druta as the 1st part. Remaining (*n*-2) beats have F(*n*-2) distinct possibilities
    - Sum gives F(*n*)
  - Pingala (500 BCE) &c already seems to be familiar with these
  - Often called “Fibonacci” numbers
    - Fibonacci wrote (about 1202) a book using “Arabic” texts that discussed Indian mathematics
    - Name given only in c. 1870's by Lucas who proved 2^127 -1 is prime using these numbers

(See Knuth, Art of Computer Programming, Vol1. p. 79-80)

How many rhythms possible using k short syllables द्रुत and n-k long syllables लघु?

- मेरु(द्रु,ल)=मेरु(द्रु-1,ल) + मेरु(द्रु,ल-1) (time not fixed)

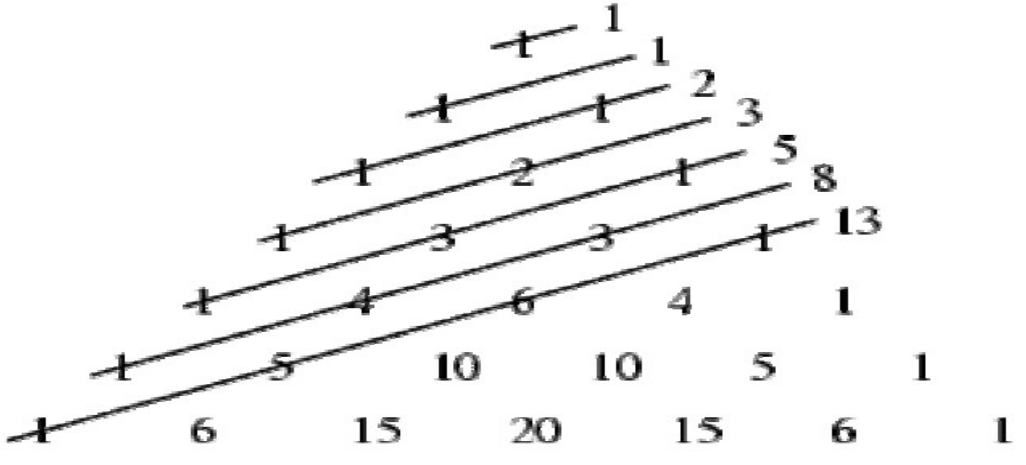
Pingala solves this problem about 500 BCE: "mountain of jewels" (wrongly called Pascal's triangle)

The solution to this problem is the combination of n things taken k at a time, or "n choose k." (n=द्रु+ल, k=द्रु)

- Above same as ${}^{n}C_{k} = {}^{n-1}C_{k-1} + {}^{n-1}C_{k}$

A "shallow diagonal" in the mountain/triangle connects the two problems, because the sum along that type of diagonal is a Fibonacci number.

- Can extend the problem to poetry with three types of syllables, which gives a three-dimensional mountain of jewels.
  - मेरु(अ,द्रु,ल)



परे पूर्णमिति ॥ ३४ ॥

अनेन एकद्व्यादिलघुक्रियासिद्ध्यर्थं यावदभिमतं प्रथमप्रस्तारवत् मेरुप्रस्तारं दर्शयति। उपरिष्टादेकं चतुरस्रकोष्ठं लिखित्वा तस्याऽधस्ताद्द्विभयतोऽर्द्धनिष्क्रान्तं कोष्ठकद्वयं लिखेत्, तस्याप्यधस्तात्त्रयं, तस्याप्यधस्ताच्चतुष्टयमेवं यावदभिमतं स्थानमिति मेरुप्रस्तारः।

Halaayudha's vritti

वर्णमेरुः।

				१					
एकाक्षरस्य प्रस्तारः	...			१	१				... २
द्व्यक्षरस्य	„			१	२	१			... ४
त्र्यक्षरस्य	„		१	३	३	१			... ८
चतुरक्षरस्य	„		१	४	६	४	१		...१६
पञ्चाक्षरस्य	„	१	५	१०	१०	५	१		३२
षडक्षरस्य	„	१	६	१५	२०	१५	६	१	६४
सप्ताक्षरस्य		१	७	२१	३५	३५	२१	७	१२८
अष्टाक्षरस्य		१	८	२८	५६	७०	५६	२८	८	१	२५६

Narayana Pandita (1356)

- Ganita Kaumudi
  - Generalizes F(n) to sum of the last *q* terms. eg.
  - “A cow gives birth to a calf every year. The calves become young and they begin giving birth to calves when they are three years old. Tell me, O learned man, the number of progeny produced during twenty years by one cow.”
  - Let V(n) be number of cows incl the 1st mother cow at n years. Assume at starting time (0th year), a calf
  - For n>3, V(n)= 1+V(n-3)+V(n-4)+...+V(1)+V(0)+3
  - For n<=3, V(0)=1+1, V(1)=3, V(2)=4

Two types of meters (vrtta)

- Varna: number of matras (laghu/guru: 0/1) fixed
  - Leads to different types of “Talas”, each tala may be of different duration for each cycle
  - Prastara (Combinatorial): 2^k for k matras
  - Karnatic music uses 1011, 101, 01, 100, 1100, 1,...
    - No 0, 00, *10*, 11, 000, *001*, *010*, *011*, *110*, 111, 0000, *0001*...
    - 0: 2 beats, 1: 3/4/5/7/9 beats
    - Also, talas with anudruta (1 beat) not given here
- Matra: number of beats fixed. The prob: how many patterns possible?
  - Leads to different types of “Talas” but each one having the same duration for one cycle
  - Number of talas given by “Fibonacci” numbers
    - F(1) =1; F(2) = 2; F(3) = 3; F(4) = 5; F(5) = 8

Binary Notation!

Table 1. ***The Piṅgala Mapping***

m	000
y	100
r	010
s	110
t	001
j	101
bh	011
n	111

Prastaara

Recursion formulation:

Pingala's alg generates the entire list of n-syllable patterns from the list of (n − 1)-syllable patterns

Suutra 8.20 dvikau glau: expansion of one syllable has two elements (a long syllable and a short syllable): base case

Suutra 8.21 misrau ca: the expansion of two syllables is the one-syllable expansion "mixed with itself."

Suutra 8.22 pruthagla mishrah: the expansion of 3 syllables

Suutra 8.23 vasavastrik: repeat for higher?

Iteration formulation:

Kedara Bhatt's algorithm gives procedure to get from one pattern on the list to the next

Nashtam

- 8.24 l-arddhe
- 8.25 sa-eke-ga
- These two sutras recover the nasht (lost or corrupted) row.
- Given a row index, get guru-laghu combination or the binary combination.
  - Write L (Laghu), if index number divisible by 2
  - Else, write G (Guru), add 1 to the index number and then halve it.
- recursive algorithm: given an index I, remove the first syllable of the pattern, to get pattern for index I-1
- Index(I-1) = Index(I /2) if I is even or Index(I+1)/2 if I is odd

Other probs considered

- Given an index number, find laghu/guru structure
  - eg. 4^{th} number with 6 beats
- Given a laghu/guru structure, find the index
- Also give first known example of “Memory Wheels”
  - Later called as “de Bruijn sequences”
  - sequence of letters drawn from some alphabet such that every combination of n letters occurs exactly once if we allow wraparound

Sqroot(2) by शूल्बकार (बौधायन, ...)

प्रमाणं तृतीयेन वर्धयेत्, तच्चतुर्थेन, आत्मचतुस्त्रिंशोनेन, सविशेषः।

$$\sqrt{2} \approx 1 + \frac{1}{3} + \frac{1}{3 \times 4} - \frac{1}{3 \times 4 \times 34}.$$

1.4142157 computed value with this approx
1.4142136 the actual value

- Connection with Vedic altars
  - Seidenberg

Sine Table using Aryabhata's (499AD) scheme!

The two shlokas gives R sin(i*225') for i=1..24 (225'=3.75 degrees)
(where R is radius 3438) in terms of differences with prev values

मखि भखि फखि धखि णखि ञखि

225, 224. 222, 219, 215, 210,

ङखि हस्झ स्ककि किष्ग श्घकि किघ्व ॥

205,199, 191, 183, 174, 164,

म: 25 ख: 2 ि: 100 => 25+ 2*100 = 225
Sin (225') = 225/3438 where Radius=3438
(भ: 24, फ: 22, ध: 19, ण:15, ञ: 10, ङ: 05)
Sin (450')= (225+224)/3438;
Sin (675')= (225+224+222)/3438...
ह: 100 स्:90 झ:9 => 100 + 90 + 9 =199
स्:90 क:1 क:1 ि:100 => 90 + 1 + 1*100 = 191
क:1 ि:100 ष:80 ग:3 => 1*100 + 80 + 3 = 183
श्:70 घ:4 क:1 ि:100 => 70+4+1*100 = 174
क:1 ि:100 घ:4 व:60 =>1*100+4+60 =164

घ्लकि किग्र हक्य धकि किच
स्ग श्झ ङ्व क्ल ष्ट फ छ कलार्धज्याः ॥

154, 143, 131, 119, 106, 93, 79, 65, 51, 37, 22, 7 are the half-chords

Bhaskara I's sine approx formula (c.650)

- sin x = 4x(180-x)/(40500-x(180-x))
  - x in degrees
  - but how arrived?
- consider sin(x) = f(x)/g(x)
  - As an approx, let both f,g be 2nd order in x
  - Also require sin(x)=sin(180-x) =>
  - invariant if x replaced by 180-x
  - f,g exprs use x(180-x)
  - So sin(x) = (a+bx(180-x))/(c+dx(180-x))
- max error 1.8%
  - But this is not good enough for Indian computational astronomy
  - Madhava (c. 1390) devised power series for sin and cos for better accuracy

Good Approximations for π!

- Madhava series
  - C= 4d[1 – 1/3 + 1/5 – 1/7 +... -1/(p-2) +...]
- Approx infinite series by

C1= 4d[1 – 1/3 + 1/5 – 1/7 +... -1/(p-2) + 1/S1]

Next approx by

C2= 4d[1 – 1/3 + 1/5 – 1/7 +... -1/(p-2) +1/p – 1/S2]

- What is the form of S1 and S2 that will be highly convergent?
- If C1=C2=> 1/S1 = 1/p – 1/S2 => 1/S1 + 1/S2 = 1/p
- Yuktibhasa commentary: best if S1=2p-2, S2=2p+2
  - Error(p) = 1/S1 + 1/S2 – 1/p
- 1st iterate: 1/(2p+2). *(Π correct to 1 dec place if p = 99.)*
- 2nd iterate: 1/((2p+2) + A/(2p+2)). Min. err if A = 4. *(5 dec places)*
- 3rd iterate: 1/((2p+2) + 4/((2p+2)+ B/(2p+2))). Min. err if B = 16 *(8 dec places)*
- 4th iterate: 1/((2p+2) + 4/((2p+2)+ 16/((2p+2)+ C/(2p+2))). Min. err if C= 36 *(10 dec)*
- 5th iterate: D= 64 *(correct to 17 places: Sadratnamaala)*

Iteration (from Bailey/Borwein 2011)

- Bakshali Manuscript
  - 1st known “quartically” convergent alg for sqroot
  - each iteration approximately quadruples the number of correct digits (provided all iterations performed with full precision)
  - Such algs used since '85 for computing value of π (eg. Borweins)
- “In the case of a number whose square root is to be found, divide [the difference from the square of the approximation] by the approximate root; multiply the denominator of the resulting [fraction] by two; square [the result]; halve it; divide it by [the first approximation]; subtract [from the first approximation]; [the result is] the refined root.” [Translation].
-

$$a_n = \frac{q - x_n^2}{2x_n}, \quad x_n + a_n, \quad a_n^2, \quad \frac{a_n^2}{2}$$

$$\frac{a_n^2}{2\left(x_n + a_n\right)}, \quad x_n + a_n - \frac{a_n^2}{2\left(x_n + a_n\right)}$$

Here is one application in the Bakhshali manuscript [11, pg. 232–233].

Problem 1 *Find an accurate rational approximation to the solution of*

$$3x^2/4 + 3x/4 \quad = \quad 7000 \tag{1}$$

(which arises from the manuscript's analysis of some additive series).

Answer: $x = (\sqrt{336009} - 3)/6$. To calculate an accurate value for $\sqrt{336009}$, start with the approximation $x_0 = 579$. Note that $q = 336009 = 579^2 + 768$. Then calculate as follows (using modern notation):

$$\begin{aligned} a_0 \;=\; \frac{q - x_0^2}{2x_0} \;&=\; \frac{768}{1158}, \qquad x_0 + a_0 \;=\; 579 + \frac{768}{1158}, \\ \frac{a_0^2}{2(x_0 + a_0)} \;&=\; \frac{294912}{777307500}. \end{aligned} \tag{2}$$

Thus we obtain the refined root

$$x_1 \quad = \quad x_0 + a_0 - \frac{a_0^2}{2\,(x_0 + a_0)} \;=\; 579 + \frac{515225088}{777307500} = \frac{450576267588}{777307500} \tag{3}$$

(note: This is 579.662833033259038 41 . . ., which agrees with $\sqrt{336009} = 579.662833033134874 98\ldots$ to 12-significant-digit accuracy).

The manuscript then performs a calculation to check that the original quadratic equation is satisfied. It obtains, for the left-hand side of (1),

$$\frac{507533837627467432 71936}{725048339467500 0000}, \tag{4}$$

Figure 3: Fragment of Bakhshali manuscript with a portion of the square root calculation mentioned in Problem 1. For example, the large right-middle section corresponds to the fraction $\frac{5075338376274674327 1936}{725048339467 5000000}$ in Formula (4). Graphic from [11, pg. 574].

which, after subtracting the correction

$$\frac{21743271936}{725048339467 5000000}, \tag{5}$$

gives,

$$\frac{5075338376272500000000}{725048339467 5000000} = 7000. \tag{6}$$

Iteration	Value	Relative error
0	579.000000000000000000000000000...	1.143×10^{-3}
1	579.662833033259038411439488233421...	2.142×10^{-13}
2	579.662833033134874975589542464552...	2.631×10^{-52}
3	579.662833033134874975589542464552...	5.993×10^{-208}
4	579.662833033134874975589542464552...	1.612×10^{-830}
5	579.662833033134874975589542464552...	8.449×10^{-3321}
6	579.662833033134874975589542464552...	6.371×10^{-13282}
7	579.662833033134874975589542464552...	2.060×10^{-53126}

Table 1: Successive iterations of the Bakhshali square root scheme for $q = 336009$ and $x_0 = 579$.

“Reverse Polish”

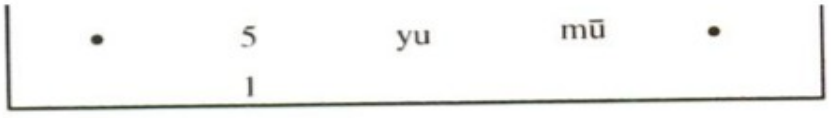
Table 2 Some Indian algebraic symbols

Current notation	Indian notation	Explanation
$A + B$	A B yu	yu from yutam, = added
$-A$	A + (BM), À (later)	+ = less by, probably from letter for kṣa in kṣaya, or *ṛṇa*.
$A \times B$, AB	A B, A B gu	gu = multiplied, from ganita
$A \div B$	A B bha	A, B dividing (or divisor).
$\sqrt{A}$	A mū	m ū = root taken, from mūlam
x	•, yā, nīla, kālaka etc.	General unknown variable
A, integer	A1	Consistent with notation for fraction below
$\frac{A}{B}$, fraction	A B	Like modern notation, but without dividing line
Ax^2	yā va A	va = squared, from varga

Source: Hayashi (1995), Datta and Singh (1962)

Here is the Indian way of writing the equation

$$\sqrt{x+5} = y$$



Notation...

BRAHMAGUPTA -PRTHUDAKASVAMI NOTATIONS

Equation with two unknowns :

yā a kā b rū 0
yā 0 *kā* 0 *rū c.*

stands for

yā (*a*+0), (plus) *kā* (*b*+0) (gives) (0+*c*)

or $ax + by = c$

with *yā = yavat-tavat, kā = kālaka*

Continued Fractions/ Sq Roots

- x^2-bx-c=0
  - x=b+c/x= b+c/(b+c/x)...
- x^2-Ky^2=1: solution of
  - (x-sqrt(K)y)(x+sqrt(K)y)=1
  - If exists r st r^2 + abs(r) <= K,
    - x/y – sqrt(K) < 1/(2y*y)
  - Consider 1766319049^2 -61*226153980^2=1
    - Solved by chakravala
  - Hence, sqrt(61) approx 1766319049/226153980
    - 7.81024967591(approx)
    - 7.81024967591(exact value)! 11 places.

kuttaka

- Solve ax-by=c
  - Arises naturally to convert from lunar to solar calendars &vv
- If (p,q) is a solution for (x,y), then any solution (p+i*b, q+i*a) is also a solution:
  - ap-bq=c
  - a(p+i*b)-b(q+i*a) = ap-bq=c
- Keep “pounding” till simple form of ax-by=c whose solution can be given by inspection
- Solve 195x-221y=65.
  - 221=195+26 => 195x'-26y=65 where x'=x-y
  - 195=26*7+13 => 26x'' + 13 x' =65 where x'' = 7x'-y
  - 2x''+x'=5 => x''=2, x'=1 => y=5 => x=6

Astronomy!

Beginning of kalpa (when sun, planets collinear)

- Sun has made some rev + 1000 days
  - 1096 days for 3 revs
- Moon some rev + 41 days
  - 5 revs in 137 days
- Mars some rev + 315 days
  - 1 rev in 685 days
- Mercury some revs + 1000 days
  - 13 revs in 1096 days
- Jupiter some revs + 1000 days
  - 3 revs in 10960 days
- Saturn some revs + 1000 days
  - 1 rev in 10960 days

Find number of days since beginning of kalpa!

Answer: 11960 days

Aryabhata Algorithm (Kak)

- Find x so that
  - x mod d1= x1,
  - x mod d2 = x2 = c+x1.

Probably inspired by investigations in continued fractions

- d2 = q.d1+r1, r1<d1
- d1=q1.r1+r2, ...
- r(k)=q(k+1).r(k+1) + 1
- x= a.d1+x1 = b.d2+x2 => find a,b so that a.d1-b.d2=c
- Work backwards to get the solution

Find x (inverse) so that ax mod b = 1

- Extended Euclidean alg finds ax+by=1, given a, b

Aryabhata Remainder theorem

- Inferred from AA
- More sophisticated than Chinese Remainder Th.

x mod 63 = x1; x mod 100 = x2 and x2-x1=70

Do the alg to get the following q coeffs on the leftmost column:

- 1 1 1 1 1 1 1890
- 1 1 1 1 1 1190 1190
- 1 1 1 1 700 700
- 2 2 2 490 490
- 2 2 210 210
- 1 70 70
- 70 70
- 0

a=1890 mod 100 = 90, b=1190 mod 63 =56

x=90*63+ x1

100/63=[1,1,1,2,2,1,3]. Convergents p_n/q_n: $p_nq_{n-1}-p_{n-1}q_n=(-1)^{n-1}$

27/17 penultimate=> 100*17-63*27=-1

Brahmagupta

- if (a,b) are solutions of Dx^2+k=y^2 and
- if (a',b') are solutions of Dx^2+k'=y^2

1: (ab' +/- a'b, bb' +/- Daa') are solutions for Dx^2+kk'=y^2

2: if a'=a, b'=b, then also (2ab,b^2+Da^2) for Dx^2+k^2=y^2

- if (a,b) are solutions of Dx^2+k=y^2

3: if k|a and k|b, then also (a/k, b/k) for Dx^2+1=y^2

- Brahmagupta's identity: $(x_1^2-N*y_1^2)(x_2^2-N*y_2^2) = (x_1x_2+Ny_1y_2)^2-N(x_1y_2+x_2y_1)^2$
- Hence, any solution avlbl $x^2-N*y^2=k$ for any k can be combined with $(m,1,m^2-N)$ to produce an another eqn with a different k.
- Consider x*x-61y*y=1. Use Bhaavana: (x*x-61y*y)^2 - 61(2xy)=1
- Keep using kuttaka (``pulverizer'') procedure to get a new eqn till k becomes 1 (which gives the solution); it always terminates.

Bhaskara II: simpler ones!

- $8x^2+1=y^2$
  - By inspection, x=1, y=3
  - Using eq.2, (6,17) also
  - Using eq.1. (1,3) & (6,17), also (35,99)
- $11x^2+1=y^2$
  - Aux. Eqn: $11x^2-2=y^2$. Solution: (1,3)
  - Use eq.2, $11x^2+4=y^2$ has sol. (6,20)
  - Rewrite (x,y) as (2x,2y) => $11.4x^2+4=4y^2$
  - $11x^2+1=y^2$ has sol. (3,10)
  - Use eq.2, also (60,199), etc.
- Brahmagupta's method needs to find aux eqn of form $Dx^2+k=y^2$ where k +/- 1, 2, 4
- Bhaskara fixed this with a better technique (chakravala)

chakravaala

- solution of equations (such as $x^2 = N*y^2+1$) using Kuttaka and Chakravala methods (iterative methods).
- Jayadeva (9th century) and Bhaskara II in 1150 CE offered the first complete solution to the equation, using the chakravala method to find (for the N = 61 case): x = 1 766 319 049 and y = 226 153 980.
- Same issued as a challenge by Fermat many centuries later to European mathematicians (again without revealing the sources)
  - first “solved” in Europe by Brouncker in 1657–58 in response to challenge
- Remarkable that within a year it was solved without any expl of the method! Chakravala method was systematically explained in the Indian tradition.
- Chakravala method "the finest thing achieved in the theory of numbers before Lagrange."

x*x - 61 y*y=1

- Start with x*x-61 y*y=k
- (x,y,k)=(8,1,3) is a solution
- (m,1,m*m-61) is also a solution trivially.
- Use Brahmagupta's identity:
  - $(x_1^2-N*y_1^2)(x_2^2-N*y_2^2) = (x_1x_2+Ny_1y_2)^2-N(x_1y_2+x_2y_1)^2$
  - (8m+61)^2-61(8+m)^2=3(m^2-61)
  - Choose m =7 so that (8+m) div. by 3 and RHS small
  - 117^2-61(15^2) = 3(-12)
  - Dividing by 3*3=> 39^2-61(5^2)=(-4). So, we now have (39,5,-4)
  - (39m+61*5)^2-61(5m+39)^2=(1-61*25)(m^2-61)
  - Let m=9. 656^2-61.84^2= -1524(20) => 164^2-61.21^2=-1905. So (164,61,-1905)
- Difficult to figure out value of m to be used...
- Brahmagupta's solution: to solve Dx^2+1=y^2, find an auxiliary eqn Dx0^2+k=y0^2 where k=+/- 1,2,4.
- Can now use Bhavana to solve!

Bhaskara's identity

- Consider Dx^2+1=y^2
- Let F(x,y) = Dx^2-y^2
- h0=F(x0,y0). Let x0,y0 be coprime.
- (x1,y1,h1)=((mx0+y0)/h0, (Dx0+m0y0)/h0, (D-m0^2)/h0)

=> Dx1^2-y^2= -h1 (by algebra)

- For x1 to be an integer, find an m so that h0|mx0+y0
  - y1 and h1 will also be! Also, x1,y1,h1 mutually coprime!
- Also, always choose m < sqrt(D) < m + h
- Initially x0=1,y0=floor(sqrt(D))

Bhaskara II: 61x^2+1=y^2

- x0,y0= (1,7). h0=61-49=12
- 12|m+7 => m=5
- x1,y1,h1=1,8,3
- x2,y2,h2=5,39,4
- 61x2-4=y^2. Rewriting (x,y)= (2x,2y) => (5/2, 39/2) is a solution for 61x^2-1=y^2
- Use eq.2, (195/2, 1523/2) a sol. for 61x2+1=y^2
- Use eq.1, betw (5/2, 39/2) and (195/2, 1523/2) to get
- (3805,29718) solves Dx^2-1=y^2
- Now use eq.2, to get (226153980, 1766319049) as solution!

Brahmagupta's identity

- if a, b, c and d are whole numbers and n is also a whole number, then the product

  $(a^2 + nb^2) \times (c^2 + nd^2)$

  is also of the form $e^2 + nf^2$ where e and f are whole numbers
- if a number is represented by a certain quadratic expression (QE) and it is multiplied by a number represented by a second QE, when can the resulting product will be always represented by a certain third QE? Gauss classified all such triples of QEs: Gauss composition (long proof of 20p!)
- Manjul Bhargava gave a “elegant” proof instead using corners of a Rubik cube in PhD thesis

Logic

- Deduction vs Induction
- Liar paradox
- Universals
- Grue paradox

Quantum...

Despite their radically different worldview, shortly after their publication it was shown that Matrix Mechanics and Wave Mechanics are mathematically identical. In fact, Schrödinger was one of the people who did the proof.

Despite their formal equivalence, there seems to be more than just logic involved in the interpretation of the mathematics. For example, Heisenberg wrote:

"The more I ponder the physical part of Schrðdinger's theory, the more disgusting it appears to me."

while Schrödinger wrote:

"If one has to stick to this damned quantum jumping, then I regret ever having been involved in this thing."

F. Theodor Stcherbatsky, Buddhist Logic, Vol I, pg 83

In the 5th century of the current era, there was a bitter argument in India between the Sankhya Hindus and the Buddhists about the nature of Universal Flux. Debates were held which lasted for days, and would attract huge crowds. According to Buddhists:

The phenomena consist of an infinity of discrete moments following one another almost without intervals... There is no matter at all, flashes of energy follow one another and produce the illusion of stabilized phenomena. The universe is a staccato movement. [Heisenberg?]

while according to the Hindus:

The phenomena are nothing but waves or fluctuations standing out upon the background of an eternal, all-pervading undifferentiated Matter with which they are identical. The universe represents a legato movement. [Schrodinger?]

F. Theodor Stcherbatsky, Buddhist Logic, Vol I, pg 83

Udayana: Predecessor of Cantor?

Logical Proof

The great logician Udayanâcārya, a specialist in ontological arguments, arrived in Puri to visit Lord Jagannātha, but found the temple closing for the day. In anger he addressed the god:

You're so drunk on wealth and power
that you ignore my presence.
Just wait: when the Buddhists come,
your whole existence
depends on me.

aiśvarya-mada-matto 'si mām avajñāya vartase
upasthiteṣu bauddheṣu mad-adhīnā tava sthitiḥ

Hindus (आत्मवादि) vs Buddhists (अनात्मवादि)

- उदयन needs refutations for (Buddhist ज्ञानश्रीमित्र's) assertions below
  - All existent things are momentary (so no आत्म!)
    - So no universals! Only particulars!
    - Universals विकल्प (conceptual constructions)!
  - No objects external to our judgements
  - No difference between an object and its qualities (properties)
  - No आत्म perceived
- Udayana उदयन theory of restrictive universals
- Udayana/Gangesa theory of induction
  - Continuity and predictability of the “real world”

Tarkasamgraha & “De Morgan Laws”

- Logical Connectives: Tarkasamgraha touches on the meaning of conjunctive and disjunctive absence, i.e. absence of (both A and B) (ubhayābhāva) and absence of (either A or B) (anyatarābhāva).
- Also, an awareness that two pots are absent is consistent with an awareness that one pot is present (and the other absent).
- Ingalls shows (1951:p 63) Nyāya recognised the validity of 2 general equations:
  - absence of (both A and B) = (absence of A) or (absence of B), &
  - absence of (either A or B) = (absence of A) and (absence of B).
- Same as “De Morgan's Laws”: ¬(A & B) ≡ ¬A ∨ ¬B, and ¬ (A∨B) ≡ ¬A & ¬B.
- Example of the second law: consider Mathuranātha's remark (1951:p66) that a ‘heap’ of (specific) absences (abhāva-kūṭa) is equivalent to a generic absence. A place which is the locus of generic absence of fire is a place at which every particular fire is absent, and conversely, if every specific fire is absent, then fire is generically absent. Thus the ‘heap’ (conjunction) of specific absences of fire, is equivalent to the absence of any fire at all (the disjunction of specific fires).

(from Jonardan Ganeri, Stanford Philosophy Ency.)

An example of Indian reasoning: Barahima

- From Persian/Arab accounts in last millennium
- “No need for prophets”
  - If prophets' words correspond to reasoning=> no need for prophets
  - If prophets' words do not correspond to reasoning=> no need for prophets again!
    - The Barāhima's Enigma: A Search for a New Solution: Binyamin Abrahamov, Die Welt des Orients, Bd. 18, (1987), pp. 72-91

Unnameability (~“Liar Paradox”)

- Can anything be named?
  - What limits exist?
- Nyaya-Vaisheshika
  - “whatever is, is knowable and nameable”
- No so: Buddhists!
  - “there are things which are unnameable”
- भर्तृहरि वाक्यपदीय (5th c.) on grammar and philosophy of language
  - Relation betw words spoken and the idea carried “fixed”
    - Signifier (vaachaka), signified (vaaachya), thatness (tattvam)
    - योग शब्दार्थः
      - (काळिदास) वागर्थाविव सम्प्रुक्तौ वागर्थप्रतिपत्तये. जगतः पितरौ वन्दे पार्वतीपरमेश्वरौ
  - Relation between them cannot itself be “signified” स्वधर्मेण svadharmena
    - What is its vaachaka, vaachya, tattvam?
  - Significance Relation is unnameable! (non-computable?)
- But that names it!

Influence of Indian logic on modern logic

- In the late 18th century, British scholars began to take an interest in Indian philosophy and discovered the sophistication of the Indian study of inference, culminating in Henry T. Colebrooke's “The Philosophy of the Hindus: On the Nyaya and Vaisesika Systems” in 1824, which provided an analysis of inference and comparison to the received Aristotelian logic, resulting in the observation that the Aristotelian syllogism could not account for the Indian syllogism.
- Max Mueller contributed an appendix to Thomson's Laws of Thought (1853), in which he placed Greek and Indian logic on the same plane: "The sciences of Logic and Grammar were, as far as history allows us to judge, invented or originally conceived by two nations only, by Hindus and Greeks.”

- Indian logic attracted the attention of many Western scholars, and has had an influence on pioneering 19th-century logicians such as Charles Babbage, Augustus De Morgan, and particularly George Boole, as confirmed by his wife Mary Everest Boole in an "open letter to Dr Bose" titled "Indian Thought and Western Science in the Nineteenth Century" written in 1901
  - their acquired awareness of the shortcomings of propositional logic are likely to have stimulated their willingness to look outside the system.
  - “Think what must have been the effect of the intense Hinduizing of three such men as Babbage, De Morgan, and George Boole on the mathematical atmosphere of 1830-1865.” Mrs. Boole (1901)
- De Morgan himself wrote in 1860 of the significance of Indian logic: "The two races which have founded the mathematics, those of the Sanscrit and Greek languages, have been the two which have independently formed systems of logic."

Navya Nyaaya नव्यन्याय

- Began around eastern India and Bengal
- Anticipated some aspects of modern logic by the 16th century, such as
  - Gottlob Frege's "distinction between sense and reference of proper names" and his "definition of number"
  - Navya-Nyaya theory of "restrictive conditions for universals" anticipating some of the developments in modern set theory
  - Udayana in particular developed theories on "restrictive conditions for universals" (jatibadhaka) and "infinite regress" that anticipated aspects of modern set theory.
- (according to Kisor Kumar Chakrabarti) May have even gone past current state of philosophy of logic !
  - The “grue” paradox of 1950's and its handling more than 6 centuries or earlier

- जाति as a universal
  - But should be restricted if it leads to inconsistencies
    - Set theory without “naivete”!
  - जातिबाधक: restrictions on universals eg.
    - अखन्डोपधि simple (सखन्डोपधि not complex). Hence, no negations
      - Also no relations (eg. brother)
    - Necessity or inseparability of universal wrt particular
    - No overlapping betw 2 universals; only proper inclusion
    - No universal admitted if its admission violates its essential nature
      - No विशेष shares any universal
      - If such a universal exists, then it can itself be a विशेष!
    - तुल्यता: 1-1 corresp? Intensional vs extensional?
    - अनवस्था
    - Avoid a set whose member is membership!
      - Consider the set of all membership relationships: x in y (say, S). Since x in {x}, domain of S is all entities. But such an universal does not exist
    - Atleast one member (eg. space is not a universal)

अनवस्था anavastha

- A restrictive condition in navyanyaaya: anavastha or vicious infinite regress: no universal (jati) can be admitted to exist, the admission of which would lead to a vicious infinite regress.
  - Note: Udayana does not mind infinite regress if not vicious!
    - eg. seed vs tree
- Udayana: there can be no universal of which every universal is a member
  - for if we had any such universal X, then, by hypothesis, we have got a given totality of all universals that exist and all of them belong to X
  - but X is itself a universal
  - but X cannot be a member of itself, because in Udayana's view no universal can be a member of itself
  - hence X too along with other universals must belong to a bigger universal and so on ad infinitum.
- interesting analog in modern set theory in which it is held that a set of all sets (i.e., a set to which every set belongs) does not exist.

Induction

- Induction a basic method of scientific and philosophical inquiry
- Intensive study of logic of induction in Indian philosphical thought earlier (about 2000 years!) in India than in Europe.
  - Sanskrit philosophical literature on this subject extensive.
  - Early: Chaarvaka (5th century BCE?)
  - Later: number of major Indian viewpoints: Udayana (11th CE), Jayarasi (7th CE), Prabhakara (8th CE), Dharmakirti (7th CE) and Prabhacandra (14th CE).
  - Nyāya view sophisticated, particularly the later Nyāya view as developed by Gangesa (13th CE)

- Hume: first European philosopher to recognize induction as an independent problem and discuss it elaborately (18th century)
  - despite Whewell, Herschel, and Mill, "it was left to Pierce in the late 19th century to bring out the value of hypothesis (calling it abduction and distinguishing it from deduction and induction)."
- Some traces of the link between causation and counterfactual conditionals found in Hume, but "no detailed and systematic study of them is found in any Western writing before this century. The same, further, applies to the principle of economy ... Similarly, a systematic study of inference to the best explanation is emerging only in some very recent publications" (Chakrabarti, *Definition and Induction,* p. 209)
- Still remains a hotly debated issue in contemporary philosophy.

Problem of Induction

- Some major (Western) contemporary viewpoints: Russell, Strawson, Reichenbach, Popper, Carnap
  - new riddle of induction: grue paradox

(Kishor Kumar Chakrabarti, 2010)

- Remarkable that Gangesa and others not only discussed the classical problem of induction but also anticipated the "new" problem of induction not found in Hume
- Nyāya view highly developed and defensible but whether the most defensible?
  - Against the skeptical tide, can argue that the method is secure and reliable.
- *Contemporary philosophers would profit if they engage seriously with older Indian views with an open mind*

Hume and Indian Logic of Induction

- Could David Hume Have Known about Buddhism? The Jesuit connection!?
  - Alison Gopnik (UCB) “Could Hume have known about Buddhism?” presented at the American Philosophical Assoc. Pacific Division meeting, April 2009
  - “Charles Francois Dolu, the Royal College of La Flèche, and the Global Jesuit Intellectual Network,” Alison Gopkin, Hume Studies, Vol 35, 2009
- Hume may have found the problem of induction on his own, the possibility that he had some knowledge of the existence of the problem in the Indian tradition cannot be ruled out. Gopkin says:
  - Hume at the Royal College of La Fleche in France in 1735–37 when he wrote his treatise.
  - Came into contact with Charles Francois Dolu, a Jesuit missionary, who lived there from 1723-40
    - Dolu respected for extensive knowledge of Eastern religions and scientific views
    - Firsthand knowledge of Therevada Buddhism in Siam in 1687–88
    - In India from 1688 to 1710 and carefully studied Buddhism including Tibetan Buddhism

    - Dolu had direct contact with Ippolito Desideri, a Jesuit, who visited Tibet and studied Buddhism.
    - Buddhist no-self theory and the Carvaka critique of induction age-old views very widely known in India and routinely included in Buddhist and Hindu texts. *Probable that Dolu studied them.*
  - Hume may have taken these views from Dolu and incorporated them into his philosophy
  - Hume's views about the self and induction not linked to earlier Western views
- Though the evidence falls short of complete certainty, it appears to be significant enough to warrant the tentative assumption that Hume was indebted to Indian philosophical doctrines
- Note that Jesuit's acted as "IP" scouts worldwide betw 1500? to may be 1800?
  - Jesuit Science and the Republic of Letters, ed. Mordechai Feingold (Cambridge, MA: MIT Press, 2003);
  - The Jesuits: Culture, Sciences and the Arts 1540–1773, ed. John W. O'Malley (Toronto: University of Toronto Press, 1999);
  - Louis Caruna, "The Jesuits and the Quiet Side of the Scientific Revolution," in The Cambridge Companion to the Jesuits, ed. Thomas Worcester (Cambridge: Cambridge University Press, 2008).

Puurva Paksha of Chaarvaka on Induction

Perception or observation of particulars is the only source of knowledge.

“Inductions using accepted procedures have been true or largely true, hence induction is justified”

- such an inductive justification of induction is circular, for the very question raised by Carvaka (much later by Hume) is whether regularity in the past can be the proper reason for regularity in the future

Say, predicate A (“fire”) and predicate B (“smoke”)

- A implies B?
- B implies A?

Instead of avlbl predicates, say A and B actually to be a trace of fire/smoke instances

- (vyabhichaara) Does A deviate from B?
- (vyaapti) Does A pervade B? Does B pervade A?
- (samanvaya) Is there a correlation or co-occurrence?
- (upādhi) Are there any hidden variables?

The commentary Prakāśa says: “If nondeviation could be ascertained from lack of knowledge of deviation, deviation should be ascertainable from lack of knowledge of nondeviation”.

Deviation and nondeviation follow respectively from presence and absence of adjuncts (upādhi); but the determination of the absence of adjuncts is impossible

What could be the ground for knowing that no unobservable adjunct is involved?

Vātsyāyana: Induction

- PROPOSITION (pratijñā): p has S
- GROUND (hetu): p has H.
- CORROBORATION (udāharaṇa): d has H and d has S.
- APPLICATION (upanaya): As d has H and has S, so p has H and has S.
- CONCLUSION (nigamana): p has S.

Statements in Vātsyāyana's syllogism all have the form of either particular or existential statements

(from Brendan Gillon, Stanford Ency. Of Philosophy)

Vātsyāyana: example of induction

- PROPOSITION (pratijñā): sound is non-eternal
- GROUND (hetu): because of having the property of arising
- CORROBORATION (drstaanta/udāharaṇa): a substance, such as a pot, having the property of arising, is non-eternal
- APPLICATION (upanaya): and likewise, sound has the property of arising
- CONCLUSION (nigamana): therefore, sound is non-eternal because of having the property of arising

Schayer's “Modern” Model

1) *pratijñā*	$\psi(a)$	There is fire on *a* (= on this mountain)
2) *hetu*	$\varphi(a)$	There is smoke on *a* (= on this mountain).
3) Statement of vyāpti	$(x).\varphi(x) \supset \psi(x)$	For every locus x: if there is smoke in x then there is fire in x
4) *upanaya* = statement of the *pakṣadharmatā*	$\varphi(a) \supset \psi(a)$	This rule also applies to *x*= *a* (for the pakṣa)
5) *nigamana* = statement of *sādhya*	$\psi(a)$	Because the rule applies to x = a and the statement φ(a) is true, the statement ψ(a) is true

Elementary Indian logic of induction

For any knower *S*, if *S* has a perceptual cognition, *Fx,* and then remembers the rule "Wherever there is *F*, there is *G"* as instantiated in the uncontroversial case *O*, and then perceives in *x* the same *F* as before but this time as figuring in the remembered rule "Wherever there is *F*, there is *G*," then *S* will experience an inferential cognition of the form *Gx*, provided that there is no relevant hindrance

Mohanty, *Reason and Tradition in Indian Thought,* p. 111

“grue” Paradox

[Goodman] 1946

- Suppose all emeralds observed so far are green.
  - seems to confirm that all emeralds are green and permit the prediction that the next emerald to be seen will be green.
- But now consider the concocted predicate “grue.”

“Something is grue if it has been found to be green whenever it has been observed so far or it is not yet observed and will be observed to be blue.”

- observed evidence seems to confirm that all emeralds are green also seems to confirm that all emeralds are grue.
- But then we seem to have two conflicting predictions equally confirmed by the same inductive evidence. If all emeralds are green, the next one should be green, but if all emeralds are grue, the next one should be blue.
- Can concoct an indefinite number of grue-like predicates and the same difficulty will arise in each case. ***Gangesa aware of them in 14th c.!***

Historical note

- D. H. H. Ingalls, Sanskrit Prof at Harvard Univ
  - studied first under Quine, a collaborator of Goodman
  - trained later (1938-41) in Nyāya philosophy by Kalipada Tarkacharya, Kolkata.
- Goodman may have been influenced by Nyāya ideas in coining perverse predicates like the grue but he makes no reference to the Nyāya anywhere. Still, it is possible that Goodman and Ingalls had some philosophical conversations [Kisor Kumar C.]
- [Note that Goodman may have also met B. K. Matilal (who spent a few years at Harvard but this is post 1957), a leading specialist in Nyāya philosophy trained by Taranath Tarkatirtha and Kalipada Tarkacharya, Kolkata.]

More detailed explanation

- X is grue
  - If observed before time t (in the future), it is green
  - If observed not before time t, it is blue

- 1st hypothesis: all green so far
  - Predict green in future
- 2nd hypothesis: all grue so far!
  - Predict grue in future
- At time > t
  - 1st hyp: green
  - 2nd hyp: grue (blue!)

“grue” and Gangesa

Gangesa [13th c]

- cooked-up property: “not being either the inferential subject or a negative instance”
- (disni: pakşa-vipakşa-anyatara-anyah)
- eg. not being either the hill or a lake: the latter is true of a fiery kitchen hearth that is neither the hill (inferential subject) nor a lake (negative instance)

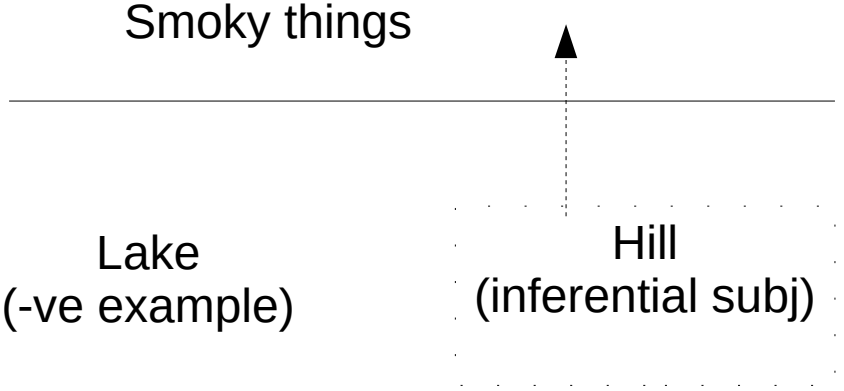
=> generalization that no fiery things are either the inferential subject or a negative instance.

At the same time disni cannot be true of the inferential subject.

Problem mainly due to the same generalization formula that permits the induction “wherever there is the probans (hyp) there is the probandum (to be proved)” also permits the induction “wherever there is the probandum there is disni”

धिष्ण्य (fireplace)

- पक्षविपक्षान्यतरान्यः यथा प्रसिद्धानुमाने पर्वत जलह्रदान्यतरान्यत्वम्
- pakşa-vipakşa-anyatara-anyah yathā prasiddha-anumāne parvata-jalahrada-anyatara-anyatvam (403–4)
- Tran. (Seventh) being different from (the pair of) either the inferential subject or the negative instances (disni)—for example, being different from (the pair of) either the hill or a lake with reference to the stock inference (of fire in the hill from smoke) (from kisor chakrabarti 2010)



fiery

non-fiery

1st hyp: fiery things are not in lower part => Hill not fiery

2nd hyp: smoke on hill => hill fiery

गंगेश (तत्वचिंतामणि)

- Insightful resolution of induction problem
- When does something that has a particular “constant” value for a long time change?
  - When an “event condition” triggers
  - In a rule based system, inference one possibility
  - So can disallow any possible “inferential subject” that is now “constant” but can change its “colours”
  - Such adjuncts are upaadhyaabhasaah (upaadhi+abhaasaah)
    - Pseudo adjuncts
- A natural problem of epistemology (“how to know something”)
  - Resource-bounded (computationally challenged?)
  - Time-bounded
- Compare with “grue paradox” which is a “totally” abstract “mathematical” formulation that hides the motivation for the problem

Induction and Machine Learning?

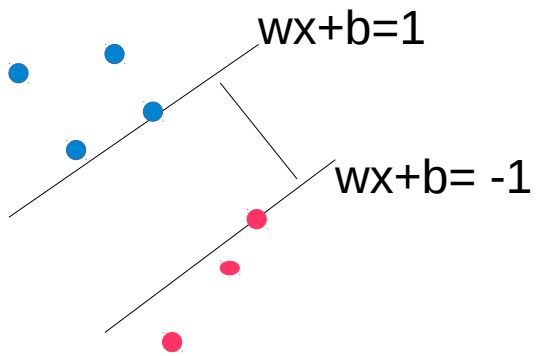
- Regression: given some data, “fit the data to a curve”
  - Overfitting?
  - Underfitting? With finite data, under determination of hypotheses?
    - Given n points <x,y>, exactly one n-order poly
      - Future data can put further constraints
      - But unknown as of now...
    - Unlimited number of poly that are > nth order poly
    - How to select one poly “now” given only n data points?
    - Best hypothesis selection? n, n+1, n+2,... poly?
    - How much data does one need before making up one's mind?
- Gangesa and “Grue” problem:
  - Predictions fit till time “t”
  - Different futures possible
    - Green remains green
    - Green becomes blue
  - Both are fine!
  - But some models are reasonable and some are not
    - grue one is unreasonable (unrealistic)

Why Study Nyaya (Kisor K C)

- While responding to the skeptical critique of induction, the Nyāya has provided
  - a powerful argument from counterfactual reasoning (CR)
  - clear arguments for defense of causality (such as the argument from the occasional nature of an effect and rejection of plurality of causes)
  - an advanced analysis of the flaw of circularity and logical economy
  - rigorous arguments for objective universals and a formidable argument from belief-behavior contradiction
- *Modern European empiricism failed to make more progress because some of these arguments remained underdeveloped and underutilized.*
  - Another reason for such lack of progress is insufficient recognition of some basic principles
  - the principle of observational credibility (OC): a factual claim that is backed by observation is preferable to one that is not.
  - the principle of general acceptability of inductive examples (GAIE, discussed in "Classical Indian Philosophy of Mind")
  - the flaw of uniqueness (asādhāraṇya)
- *Nyāya not above criticism and beyond challenge but a more advanced empiricism could emerge from a cross-cultural and comparative study of European and Indian empiricism even in the current decade!*

A similar problem in ML

- SVM: classification problem
  - Select best separating plane but that is also robust
  - Best if we can maximize separation (2/||w||)
    - Min ½ $||w||^2$ (over w and b) along with condition that each point is “above” or “below” the corresp hyperplane
    - But this overfits
  - Soft margin: if no such hyperplane exists (no clean classification)
    - Add a scaled error margin and minimize error coeff
      - Min ½ $||w||^2$ + C.sigma(e) over w,b,e



Modal logic

- In Tattvachintamani
- 4 valued logic: चतुष्कोटि (Buddhism)
- 8 valued logic: saptabhaṅgī (Jainism)

How to Preserve Texts?

- A Substantial Oral Tradition in India
- Problems of Corruption
  - Indrashatru vs Indrashatru
    - Different stress gives opp. Meanings
    - Indrashatru with stress on shatru: slayer of Indra
    - Otherwise, just an enemy of Indra
  - Nih + devatvam (no gods) vs nidratvam
  - Vedic Altar Geometry: efficacy lost if geometry wrong
- Contemporary example: saving a doc for use 1000 years later?
- Technical Approaches: coding, checksum/parity, replication

Saving Current Documents for the Next Millennium

Across time (written now 2012 and read then 3012) & space (written here on Earth & read there on Mars!)

- Say document written in Postscript/ Wordstar/ Word!
- Stored on a SCSI 36GB 15K rpm disk drive on, say, a ext3 filesystem on Linux...

What is the ***hard*** technical problem?

- Drives/device driver/filesystem/kernel/application may become obsolete
- Along with document, the associated model of device/software may need to be saved (recursion problem!)

Any current technologies useful?

- ASN.1? Virtual machines? Virtualization?!

Till today no effective solution!

Indus Script

- Yet undeciphered: meaning across time not yet accomplished
- Compare with hieroglyphics (Egyptian Rosetta stone): three scripts side by side

- What is the problem? Not enough contextual info:
  - can see the script (human "readable")
  - no mapping between symbols and phonemes
  - need interpretation of sequences of symbols
  - a problem in archeology, history, society,...

Vedas

Transmitted across atleast 3500-5000 years without differing versions

- Only one doubtful reading in Rigveda after 7000 years? (RV7.44.3)

Including exact pronunciation!

"UNESCO proclaimed the tradition of Vedic chant a Masterpiece of the Oral and Intangible Heritage of Humanity on November 7, 2003"

What "technology" used? Redundancy!

Various "pathas" of Samhita text: can recover from a corrupted text due to added redundancy: RAID-like! (Redundant Array of Indep Disks)

Pada-patha: each word in its separate form

Krama-patha: connects a word in pairs

ABCD becomes AB BC CD DE... ("2-mirroring"): 2 copies

Jata-patha: ABBAAB ("3-mirroring"): 3 copies of A, B, ...

Ghana-patha (ABBA ABCCBA ABC BCCB BCDDCB BCD...)("10x")

Metrical (similar to checksums!) & Musical

"Information dispersal"

Human Reproduction! (Oral transmission)

Use efficient "virtualizers"!

Recitation

vakya, pada, krama, jata, mala, sikha, rekha, dhvaja, danda, ratha, ghana

- संहित
- वाक्य
- पद
- क्रम: 12, 23, 34,...
- जट: 122112, 233223, 344334,...
- माल
- शिख
- रेख
- ध्वज
- दंड
- रथ
- घन: 1221123321123, 2332234432234,...
- Also क्रममाल and पुष्पमाल

पदपाठ

1,2,3

तम् । भागधेयेन । वि । मुञ्चति । प्रतिष्ठित्यै । यया । रज्ज्वा । उत्तमां । गाम् । आजेत् । ताम् । भ्रातृव्याय । प्र । हिणुयात् । निर्ऋतिम् । एव । अस्मै । प्र । हिणोति ॥ तै सं २-२-६-५ ॥

घनपाठ

1,2,2,1,1,2,3,3,2,1,1,2,3.

तं भागधेयेन भागधेयेन तं तं भागधेयेन वि वि भागधेयेन तं तं भागधेयेन वि ॥

भागधेयेन वि वि भागधेयेन भागधेयेन वि मुञ्चति मुञ्चति वि भागधेयेन भागधेयेन वि मुञ्चति । भागधेयेनेति भागऽधेयेन ॥

वि मुञ्चति मुञ्चति वि वि मुञ्चति प्रतिष्ठित्यै प्रतिष्ठित्यै मुञ्चति वि वि मुञ्चति प्रतिष्ठित्यै ॥

मुञ्चति प्रतिष्ठित्यै प्रतिष्ठित्यै मुञ्चति मुञ्चति प्रतिष्ठित्यै यया यया प्रतिष्ठित्यै मुञ्चति मुञ्चति प्रतिष्ठित्यै यया ॥

प्रतिष्ठित्यै यया यया प्रतिष्ठित्यै प्रतिष्ठित्यै यया रज्ज्वा रज्ज्वा यया प्रतिष्ठित्यै प्रतिष्ठित्यै यया रज्ज्वा । प्रतिष्ठित्या इति प्रतिऽस्तित्यै ॥

यया रज्ज्वा रज्ज्वा यया यया रज्ज्वोत्तमामुत्तमꣳ रज्ज्वा यया यया रज्ज्वोत्तमाम् ॥

रज्ज्वोत्तमामुत्तमꣳ रज्ज्वा रज्ज्वोत्तमां गां गामुत्तमꣳ रज्ज्वा रज्ज्वोत्तमां गाम् ॥

उत्तमां गां गामुत्तमामुत्तमां गामाजेदाजेद्गामुत्तमामुत्तमां गामाजेत् । उत्तमामित्युत्ऽतमाम् ॥

गामाजेदाजेद्गां गामाजेत्तां तामाजेद्गां गामाजेत्ताम् ॥

आजेत्तां तामाजेदाजेत्तां भ्रातृव्याय भ्रातृव्याय तामाजेदाजेत्तां भ्रातृव्याय । आजेदित्याऽअजेत् ॥

तां भ्रातृव्याय भ्रातृव्याय तां तां भ्रातृव्याय प्र प्र भ्रातृव्याय तां तां भ्रातृव्याय प्र ॥

भ्रातृव्याय प्र प्र भ्रातृव्याय भ्रातृव्याय प्र हिणुयाद् हिणुयात् प्र भ्रातृव्याय भ्रातृव्याय प्र

Ghana Patha

Play recording...

http://sanskrit.safire.com/audio/TS_ghana_2_2_6_5.ram

A correct recitation of a veda mantra should conform to the following six parameters

- *varNa* (letters) [taken care of by coding]
- *svara* (intonation)
- *maatraa* (duration of articulation)
- *balam* (force of articulation)
- *saama* (uniformity), and
- *santaana* (continuity)

Some observations:

- If two verbs come together, the tone changes (Panini)
- Can such properties be used in future “PUI”?
  - “Phonological User Interface” instead of GUI

Codes or Vikratis

- Kashyap/Bell formulate Krama-maala as a rate 1/4 linear block code over a finite Galois field
  - why these specific codes?
- Req
  - Preserve order of words
- Errors to be detected:
  - Add/delete of a syllable/word in a word/sentence
  - Avoid “long jumps”
    - A (in verse x) similar to B (in verse y) ;
    - Consider chanting of ...AC... ; ...BD...
    - Problem: during chanting mistakenly ...AD... or ...BC...
    - Handled by codes such as avichakra ratha (Kashyap)

RV 1.1.1. ... C ratnadhatamam (A)

- RV 1.20.1 ... E ratnadhatamah (B) D
- Chanting st A chained to C and B to E to prevent jump from C to B and E to A

Krama mala

- A_1 A_2 A_n A_n
- A_2 A_3 A_n A_{n-1}
- … … … …
- A_{n-1} A_n A_3 A_2
- A_n A_n A_2 A1

bruyat-krama – viparyasav-ardharcha-asya-adito-antatah | (line 1)

antam-chadim-nayedevam-kramamaleti-giyate || (line 2)

Word to word meaning:

Line 1: declared-krama-reversed or flipped-one half of a rik or verse-its-beginning-end

Line 2: end-and beginning-knit-*Krama-maala*-sing

Read out row by row 4n symbols; bend it after 2n and and make it into a 2x2n matrix. As it is a palindrome, every column (2n of them) has same symbol

(Note A_n A_n in the last row)

Decoding

- There are 4 ways of getting back the string if correctly recited
- If incorrectly recited, take the majority at each position of the 4 decodings to get the correct result
- Example: a b c d =>
  - a b d d b c d c
  - a b d d b c d c
- 4 readings
  - 1 a b d d b c d c
  - a b d d b c d c
  - 2 a b d d b c d c
  - a b d d b c d c
  - Also the 2 reverse

Error Detection/Correction

- Suppose 6 errors (b' <> b”, etc.)
  - a b' d' d b c' d c
  - a b d d” b” c d c”
- 4 readings:
  - a b c” d” a b' c' d a b c d a b” c d
  - Maj => a (4/4) b (2/4) c (2/4) d (3/4)
- Assumptions: 1st symbol (A1) is not incorrect, atmost 2 errors in each symbol in its 4 places, no error repeated (for each symbol, neither of its 2 error transformations same)
  - Allows 2(n-1) errors out of 4n symbols
- Practice? (Kashyap): one person 1000 verses of each vikrati
  - 10 (for 10,000 riks) x 5 (# of vikratis?) people needed
  - RV 10.191 “talks” about assemblies to get correct mantra?

Conclusion

- A positive outlook on understanding and engaging with the real world
- Theorize but attempt to reconcile with observations
- “Computational Positivism” in mathematics and computation
- Paanini set the stage?