2 years M. Sc. Program in Mathematics

(w. e. f. Academic Session 2019-20)

OUTLINE SYLLABUS

Ordinary Differential Equations

Initial value problems, boundary-value problems, variation of parameters, Cauchy-Euler equation, series solution, Bessel's equation and Bessel functions, solution of system of linear differential equations by matrix method, the fundamental existence and uniqueness theorem, Sturm-Liouville boundary value problems, Green's function, nonlinear ordinary differential equations.

Real Analysis

Sets, convergence of sequences and series, absolute convergence, limit, continuity, compactness, connectedness, Riemann-Stieltjes integral, sequences and series of functions, uniform convergence, Stone Weierstrass theorem, Lebesgue measure theory

Abstract Algebra

Groups, subgroups, cyclic groups, permutation groups, Cayley's, Cauchy's and Sylow's theorems, Rings, ideals, integral domain, polynomial rings, modules, quotient modules, cyclic modules, free modules, fields, finite fields, field extension, splitting fields and Galois theory.

General Topology

Metric space, Cantor intersection theorem, topological space, basis for a topology, subspace topology, weak topology, product topology, quotient topology, Continuous maps, continuity theorems for open and closed sets, homeomorphism, connected spaces, totally disconnected space, locally connected space, compact space, continuity and compactness, Tychonoff theorem, first and second countable spaces, normal spaces, completely normal and completely regular spaces, Tietz extension theorem, Uryshon lemma, Uryshon metrization theorem.

Mathematical Methods

Introduction, Euler equation, variational problems with constraints, geodesics and isoperimetric problems, moving end problems, Rayleigh-Ritz, Galerkin's and Kantorovich method. Integral equations of Fredholm and Volterra type, integral equations with degenerate kernels, Fredholm's theorems, eigenvalues and eigen functions of integral equations, Green's function, influence function, Abel integral equation, weakly singular kernel, Laplace transform, Fourier transform, Hankel and Mellin transform, Fox's integral equation.

Presentation and Communication Skills

English as a communication tool, basic aspects of English, vocabulary extension, fundamentals of grammar, skills of good speaking, professional application/ writing, textual organization, letter writing, circulars, notices, agenda, minutes, report writing.

Linear Algebra

Vector space, linear combination, linear dependence and independence, basis and dimension, linear transformation, null space and range space, rank-nullity theorem, change of basis, linear functional, eigenvalues and eigenvectors, diagonalisation, invariant subspaces, Jordan canonical representation, norm of a matrix, inner product space, orthogonal and orthonormal vectors, normed space, Gram-Schmidt process for orthogonalisation, quadratic forms.

Complex Analysis

Function of complex variable, analytic functions, Cauchy Riemann equation, Cauchy's integral theorem and formula, Liouville's theorem, maximum modulus principle, analytic continuation, zeros and singularities, Taylor and Laurent series, residues, Cauchy residue theorem and its applications in evaluation of real integrals, conformal transformations, bilinear transformations.

Computer Programming

Number system, integer and floating point arithmetic, expressions and operators, conditions and selection statements, looping and control structures, string processing, addresses and pointers, arrays, pointers into arrays, constants, references, structures, functions, parameters, passing by value, passing by reference, passing arguments by constant reference, recursive functions, function overloading and default arguments, classes, access control, class implementation, constructors, destructor, operators overloading, friend functions.

Functional Analysis

Normed space, Banach space and related results, subspace of Banach space, finite dimensional normed space and subspaces, linear operators, bounded and continuous linear operators, principle of uniform boundedness, boundedness and continuity of linear transformations, Hahn-Banach theorem, open mapping theorem, closed graph theorem, Inner product spaces, Schwartz and Minkowski inequalities, Hilbert spaces, relation between Banach and Hilbert spaces, projections, orthonormal basis, Riesz-representation theorem, convex sets, projection theorem, orthogonal and orthonormal systems in Hilbert spaces, characterization of complete orthonormal systems, Banach fixed point theorem and its simple applications.

Partial Differential Equations

Linear, semi-linear and quasi-linear equations, Cauchy problem, method of characteristics, nonlinear first order PDE's, complete integrals, envelopes and singular solutions, classification of second order equations, Laplace equation, fundamental solutions, maximum principles and mean value formulas, properties of harmonic functions, Green's function, parabolic equations in one space dimension, fundamental solution, maximum principle, wave equation, Duhamel's principle, methods of separation of variables for Laplace, heat and wave equations.

Mathematical Statistics

Probability, random variable, discrete and continuous distributions, theory of sampling, point and interval estimation, hypothesis testing, analysis of variance, regression

Numerical Analysis

Errors, roots of algebraic and transcendental equations, rate of convergence, iterative methods for system of linear algebraic equations, methods for eigenvalues and eigenvectors, interpolation and approximation, numerical differentiation and integration, numerical methods for differential equations.

Operations Research

Linear programming problems, simplex algorithm and its variants, duality and sensitivity analysis, transportation problems and their optimal solution, assignment problems and their solutions, queuing models and their solutions, inventory control models with and without shortage.

Course Outline of Elective Courses

Advanced Matrix Theory

Brief review of matrix algebra, echelon form, solution for a linear system, Gauss Jacobi, Gauss-Seidel and LU decomposition methods, inner product spaces, normed spaces, Gram–Schmidt orthonormalization, orthonormal basis, decomposition of a vector, eigenvalues and eigenvectors, power and inverse power methods, Q-R algorithm, eigen system of Hermitian matrix, quadratic, positive and canonical forms, reduction to canonical form, spectral radius, Gershgorin's theorem, polynomials and functions of matrices, derivatives of a matrix, discrete dynamical system, system of linear differential equations, fundamental form, reduction of a system, solutions of systems with constant coefficients.

Measure Theory

Algebras and σ-algebras, measures, outer measures, measurable sets, Lebesgue measure and its properties, non-measurable sets, measurable functions and their properties, Egoroff's theorem, Lusin's theorem; Simple functions, integral of bounded functions over a set of finite measure, bounded convergence theorem, integral of nonnegative functions, Fatou's lemma, monotone convergence theorem, the general Lebesgue integral, Lebesgue convergence theorem, change of variable formula; functions of bounded variation, differentiation of an integral, absolute continuity; Signed and complex measures, Radon-Nikodym theorem, Lp-spaces and their dual; Product measures, constructions, Fubini's theorem and its applications.

Differential Geometry and Tensors

Theory of Space Curves-The Serret-Frenet formulas, Gauss Theory of Surfaces- First and second fundamental form, examples, principal curvatures, Gaussian curvature, examples, computation of the curvature in standard spaces: Sphere, Torus, Surfaces of revolution etc. Gauss theorem, geodesics, equations of geodesics, examples, Weingarten equations, tensors

Fluid Dynamics

Basic concepts, governing equations of fluid motion, Lagrangian and Eulerian descriptions, continuity of mass flow, rotational flows, streamlines, general equations of motion, Bernoulli's theorem, compressible and incompressible flows, stream function, complex-potential, source and sink, motion of a sphere, Helmholtz's vorticity equation, navier-stokes equations, steady flow between two infinite parallel plates, through a circular pipe, energy equation, flow past a flat plate, dynamical similarity, dimensional analysis, Reynold's numbers, laminar boundary layer equations, similar solutions, dynamical similarity, thermal boundary layer equation for incompressible flow, temperature distribution in coutte flow and in flow past a flat plate, stability of flows under different cases.

Wave Propagation

Analysis of stress and strain, Mohr's circle diagram, Generalized Hook's Law, different types of symmetry, Plane waves, Principle of superposition, D' Alembert's formula, Spherical waves, Poisson and Helmholtz's formula, P and S waves and their characteristics, Reflection and refraction of plane P, SV and SH waves at an interface, Surface waves: Rayleigh, Love, Torsional and Stoneley waves, Interior structure of the Earth, Location and causes of Earthquake, Earthquake magnitude.

Continuum Mechanics

Stress, strain, principal strains, equations of compatibility, finite deformations, equations of equilibrium, generalized Hooke's law, equilibrium equations for an isotropic elastic solid, plane

strain, plane stress, Airy stress function, Cartesian coordinate solutions using polynomials, general solution in polar coordinates.

Fuzzy Sets and Applications

Basic concepts of fuzzy sets, its different types and operations, Zadeh's extension principle, fuzzy relations and fuzzy relation equations, fuzzy arithmetic, fuzzy logic, linguistic variables and fuzzy inference, Possitbility theory, fuzzy measures, probability of a fuzzy event fuzzy mapping rules, implications and approximate reasoning, decision making in fuzzy environment.

Data Structures

Course covers analysis and design of fundamental data structures. The course focuses on basic and essential topics in data structures, including array-based lists, linked lists, skiplists, hash tables, recursion, binary trees, heaps, sorting algorithms and graphs.

Multivariate Analysis

Multivariate normal distribution, moment generating function, characteristic function, moments, marginal and conditional distribution, multiple regression, Maximum likelihood estimators, Wishart distribution, Hotelling T^2 statistic, discriminant analysis.

Wavelet Theory and its Applications

Introduction to basic linear algebra, Fourier analysis, scaling function and wavelets, discrete wavelet transform, filter banks, applications to different areas, wavelet software.

Number Theory

Divisibility, GCD, LCM, prime, conjectures, Fermat and Mersenne primes, residue classes and reduced residue systems, Chinese remainder theorem, greatest integer function, arithmetic function, Mobius inversion formula, the number-of-divisors and sum-of-divisors functions, Perfect numbers, RSA Cryptosystems, primitive roots, theory of indices, quadratic residues, Legendre symbol, Euler's criterion, Gauss lemma, law of quadratic reciprocity, Finite continued fractions, periodic continued fractions and quadratic irrationals, Pell's equation, Fermat's Last Theorem, Riemann Zeta function, Euler product formula, convergence, Dirichlet L-functions.

Graph Theory

Basic graph terminology, tree, cut set, planarity and vector spaces, matrix representation of graph, graph coloring, enumeration and graph theoretic algorithms.

Advanced Numerical Methods

Newton-Raphson method for two or more variables, Graeffe's method, Hermite interpolation, cubic splines, B-splines, approximation by method of least squares, (SOR) method for linear system of equations , norms of vectors and matrices, predictor-corrector methods, finite difference methods, finite element method.

Theory of Computation

Basic concepts of deterministic and non-deterministic finite automata, regular language, context-free language, Turing machines, Church's thesis, halting problem, computability and complexity

Database-Management System

Why database, different models of database, relational database, functional dependencies and key concepts, normalization, SQL and PLSQL, transaction and concurrency, serialization.

Advanced Operations Research

Review of Linear programming problems, duality theory, simplex algorithm and its variants, sensitivity and parametric analysis, deterministic inventory models, network analysis using PERT and CPM, game theory and linear programming, multi-objective and goal programming problems, non-linear programming, quadratic and separable programming problems.

Theory of Data Science

The art of data science, machine learning, supervised and unsupervised learning, methods for function approximation, linear models, models for classification, dimensionality reduction, extracting information from news.

Linear Models and Regression Analysis

Random Vectors and Matrices**,** Grauss-Markov Linear Models**,** Normal equations and least square estimation, one way and two-way ANOVA, Simple linear and multiple regressions.

Mathematical Imaging

Digital image fundamentals, multi view geometry, line and curve drawing algorithms, 2-D and 3-D transformations, spatial image enhancements, frequency domain image enhancements, image restoration, mathematical morphology of images

Lab Courses

Lab- 1

Computer Programming Lab

1. Basic data types, constants and variables, Arithmetic operators, built-in mathematical functions. Arithmetic expressions. Logical and relational operators, scanf() and printf() functions.
2. I/O using cin and cout, simple programs, control of flow using if, if … else, goto.
3. Loops for, while and do … while, use of break, return and exit.
4. Programs for n!, e^x, sin$x$, log(1+x).
5. Arrays and strings, Sorting of arrays.
6. User defined functions, call by value/reference, default parameters, returning values.
7. Recursive functions.
8. Pointers and their applications in handling arrays and strings.
9. Structures, distance, complex, date.
10. Object and classes, Constructor/destructors, Private and public. More objects.
11. Complex class, distance class, Matrix class.
12. Operator overloading, Functions with objects, Friend functions, I/O handling in C++.

Lab-2

Numerical Analysis Lab

Practical/Lab to be performed on a computer using C++/ MATLAB.

1. To find a real root of an algebraic/ transcendental equation by using Newton-Raphson method.
2. To find a real root of an algebraic/ transcendental equation by using Successive iteration method.
3. To find a root of an equation by using Muller's method.
4. Implementation of Gauss-Elimination method to solve a system of linear algebraic equations.
5. Implementation of Gauss-Jordon method to solve a system of linear algebraic equations.
6. Implementation of Gauss-Seidel method to solve a system of linear algebraic equations.
7. Implementation of Lagrange's formula for interpolation.
8. Implementation of Newton's divided difference formula for interpolation.

9. To find differential coefficients of 1st and 2nd orders using interpolation formulae.
10. To evaluate integrals by using Trapezoidal rule.
11. To evaluate integrals by using Simpson method.
12. To compute the solution of ordinary differential equations by using Euler's method.
13. To compute the solutions of ordinary differential equations by using Runge-Kutta methods.
14. To solve two point boundary value problem by shooting and finite difference method.

Operations Research Lab

Practical/Lab to be performed on a computer using OR/Statistical packages

1. To solve Linear Programming Problem using Graphical Method
2. Solution of LPP with simplex method.
3. Problem solving using Charnes-M method (Big-M).
4. Problem solving using Two Phase method.
5. Illustration of following special cases in LPP using Simplex method
   (i) Unrestricted variables (ii) Unbounded solution
   (iii) Infeasible solution (iv) Alternative or multiple solutions.
6. Problems based on Dual simplex method.
7. Solution of Transportation Problem.
8. Solution of Assignment Problem.
9. Solution of IPP using Branch and Bound method.
10. To determine local/Relative optima of a given unconstrained problem.
11. Test whether the given function is concave / convex.
12. Test whether the given matrix is positive definite/negative definite
13. Solution of optimization problems using Karush-Kuhn-Tucker conditions.
14. Solution of Quadratic programming problem by Wolfe's method.